The Efficiency of Chinese College Admission Mechanisms: a Perspective from Matching Theory and Online Learning Algorithms

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Abstract

Since the new century, the Chinese college admission mechanism has shifted from an Immediate Acceptance (IA) style mechanism to a Deferred Acceptance (DA) style mechanism. Although DA is often regarded as a superior matching mechanism than IA, their social welfare implications are not clear. This paper defines a new property called sensitive-to-top choice, and finds that this class of mechanism generates stable matching in equilibrium. When there is uncertainty regarding the ranking of students, this paper uses no-regret online learning algorithms to model students, and study the equilibrium properties of admission mechanisms in the Chinese context. I find that DA generates slightly higher expected efficiency, while IA exhibits less volatility. A more realistic model of the current Chinese college scheme, the Parallel Acceptance Mechanism (PA) lies between DA and IA in expected efficiency. The volatility of PA lies in the middle of PA and IA when the population size is large.

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1 Introduction

Every year about 10 million Chinese high school graduates compete for 6 million seats at Chinese universities[1]. The process usually consists of College Entrance Exam (CEE, also called Gaokao), and a matching procedure, where students submit a list of preferred colleges, and each college admits students based on the submitted preferences, CEE scores, and the college quota in a province. Therefore, how this matching process is designed has profound impact on the fairness and efficiency of the talent and education resources allocation within Chinese higher education system.

Starting from the new century, the matching procedure has undergone many reforms. Most significantly, it moved away from a pre-exam preference submission Immediate Acceptance (IA) mechanism to resemble more closely a post-exam Deferred Acceptance (DA) mechanism. The most realistic model of the current mechanism is named Parallel Acceptance (PA) and studied by Chen and Kesten[11]. In fact, US public high schools in many cities and states have experienced a similar transformation. First formalized by David Gale and Lloyd Shapley[9], the deferred acceptance mechanism has been proved to have a few superior properties, and have motivated education public officials and economists to design new matching clearing house in DA styles [7][8].

Economists have recently started to reconsider the IA mechanism, as the superiority of DA turns out not to be unclear under some circumstances. When schools have coarse preferences over students and students have homogeneous preferences over schools, Abdulkadiroglu et al. showed that IA has some superior welfare properties compared to DA. When there is uncertainty involved in school preferences over students, such as when Chinese universities receive Gaokao scores – non-perfect signals of students' academic aptitudes – Wu and Zhong found that there is no significant difference in academic abilities between students admitted to a top Chinese college through IA and DA mechanisms [13].

This paper defines a class of sensitive-to-top-choice student-proposing college admission mechanisms that satisfies some desirable properties when players strategize in the preference-submission game. IA is sensitive to top choice, while DA and PA are only sensitive to top choice in a more restrictive class of admission problems. I prove that every Nash equilibrium in a sensitive-to-top-choice mechanism will generate a stable matching. For any Chinese college admission problem, I prove that any coarse correlated equilibrium will generate the unique stable matching with certainty.

A general difficulty in analysing mechanisms like IA and PA, where there is no dominating strategy, is the complicated equilibrium behaviors especially when there are a large number of players. To address this problem, this paper adopts no-regret online learning methods in the algorithmic game theory literature as students playing the preference-submission game to compare different mechanisms in the Chinese context. We know that no-regret online learning algorithms will converge to a coarse correlated equilibrium, thus generating the only stable outcome by my previous theoretical result. Finally, I manipulate simulation parameters to study pre-exam IA, DA, and PA outcomes with online learning algorithms under different levels of risk aversion, exam variances, school capacities and population sizes. I find that IA yields a slightly lower efficiency level than DA while exhibiting lower level of variance. PA lies in between IA and DA in expected performance. The variance of PA is between IA and DA when population size is large.

2 An Overview of Student-Proposing Mechanisms and Their Properties

2.1 Properties of Admissions Mechanisms

We fist follow Roth [2] to characterize some desirable properties of an admission mechanism.

First, it's intuitive to desire some notion of fairness in school admissions. Formally, a matching μ generated by a mechanism is ***fair***, if a student prefers another school to the school they are assigned to, there should be no students in the preferred school who has a lower priority than them. In a more general context, we allow school to also have preferences over students, but in the Chinese case we force the colleges to have a homogeneous preference that is induced by the natural ranking of CEE scores. A fair mechanism is also said to be ***stable***.

Second, we might also want to match talent and education resources to achieve efficiency. A matching is ***Pareto efficient*** if there is no switching school assignment for two students such that the switching would make one of them strictly better off and the other at least weakly better off. First note that this Pareto efficiency only cares about students' preferences. In addition, we only care about the ordinal preferences but not the cardinal one, which means that we do not care about the intensity of student preferences. We will discuss cardinal preferences later.

Finally, since in the school admissions settings, we ask students to submit their preferences, we would like the preferences submitted to be reflective of their true preferences, all else equal. We say a mechanism is ***strategy-proof*** if when every one else is submitting their true preferences, it is in a student's best interest to submit their own true preferences as well.

An interesting property that would have more importance in a score-based student priority setting is ***respecting improvements***, which means that an improvement in a student's score should at least allow them to enter the same college. This is explored in the Turkish college exam context by Balinski and Sonmez[3].

2.2 The Immediate Acceptance Mechanism and The Deferred Acceptance Mechanism

The immediate acceptance mechanism (IA), often referred to as the Boston mechanism, named after the Boston Public School admission process until 2005, is a popular mechanism around the world in college and high school admissions. We follow Pathak [4] to describe the mechanism:

1. Step 1: Each school considers students who list it as the first choice, and assigns seats to those students in the order of the school priority (exam score in the Chinese case) until the number of admitted students equals to the school's capacity. The school rejects any remaining students.

2. Step k: Each school considers students who have been rejected in the previous round and list it as their k^{th} choice. The school then assigns seats to those students in the order of the school priority until the number of admitted students equals to the school's capacity. The school rejects any remaining students.

The process terminates when every school is at full capacity or when every student has been assigned a seat. The Boston mechanism is trivially Pareto efficient, but it's not stable or strategy proof. In fact, there were lots of sophisticated Boston parents gaming the mechanism. They might be afraid that their kids did not have high enough priority at a popular school, so they listed a less popular school as their top choice. Also, since the mechanism prioritizes student's first choices compared to their overall ranking, there could also be cases where students with lower scores being admitted to better colleges in the Chinese case. The lack of stability and strategy-proofness have been identified both in a game theoretical context [5] and in experimental settings [6].

The above lack of good properties of the Boston mechanism pushed policy makers in Boston and New York to reform their public high school admission processes. Policy makers invited economists to help redesign the admission mechanisms in the two areas, and greatly improved the matching stability, strategy-proofness, and efficiency by switching to a deferred acceptance mechanism (DA)[7][8]. In the first year that the new mechanism was adopted in New York, DA matched 20,000 more students with high schools on their first choice list, a 40% increase from the previous year. We will describe this deferred acceptance mechanism following Gale and Shapley, who first proposed it in 1962[9]:

1. Step 1: Each school considers students who list it as the first choice, and tentatively

assigns seats to those students in the order of the school priority (exam score in the Chinese case) until the number of tentatively admitted students equals to the school's capacity. The school rejects any remaining students.

2. Step k: Each school considers students who have been rejected in the previous round and list it as their k^{th} choice. The school then assigns seats to those students and also those who are tentatively admitted in the order of the school priority until the number of newly tentatively admitted students equals to the school's capacity. The school rejects any remaining students.

The process terminates when every student is tentatively admitted at one school, or when every rejected student has no more schools on their list. When the process terminates the tentative assignment becomes final. As shown by Roth in 1982, this mechanism is stable, strategy proof, and Pareto dominates any other stable mechanism for the students, which means every student at least weakly prefers their assignment under DA than any other stable matching [2]. It's interesting to note that if we consider schools also as agents in the admission process, there exists no stable mechanism where both students and schools will be truthfully reporting their preferences. We now turn back to the Chinese college admissions problem.

2.3 The Parallel Acceptance Mechanism in China: a Reconciliation

Before 1952, the college admission process was decentralized in China and suffered a lot of congestion and coordination issues. In order to centralize higher education and also to solve the above problems, the National Ministry of Education introduced the College Entrance Exam, or Gaokao, as an institution that exists till today. Before 2003, every province had been using an IA style mechanism [10]. A lot of fairness and stability issue emerged. As an example, my grandfather went to Shanghai Jiaotong University in 1963, and he once told me that his high school classmate who scored lower than him in Gaokao went to Tsinghua University, which is regarded as one of the top two colleges in China. This happened because the different ways they filled out their college preference lists. Stories like this are not uncommon before 2003.

To tackle the new issues, Chinese provinces started to move towards a parallel acceptance (PA) mechanism in 2003, and PA was quickly adopted by every province in subsequent years due to wide popularity among students and parents. In PA, every student's first e choices on the submitted preference list are considered in a DA fashion, but after the first e rounds the DA tentative assignment becomes permanent, so the first e rounds serve as one IA round in some sense. The remaining students engage in a new DA mechanism. Following Chen and Kesten [11], we can parameterize the family of PA mechanisms by a vector $(e_1, \ldots, e_n)$ where at the end of $x_i = \sum_{k=1}^{i} e_i$ round, the tentative assignment becomes permanent. Intuitively, we can say that each e_i schools as in the i^{th} DA group. PA family mechanisms are proven to be more stable and strategy-proof than the IA mechanism in theory, while less so than the DA mechanism [11]. An experimental study done by Chen and Kesten to compare IA, DA, and PA found that the stability and strategy-proofness of PA are consistently sandwiched between IA and DA as predicted by theory [1].

An interesting question remains: why not simply adopt a DA mechanism? In fact, in Chen and Kesten's study the efficiency of IA is often higher than that of DA. In a theoretical paper by Abdulkadiroglu et al., they reconsider the Boston mechanism, and find that when the students have identical preferences (which is often true as some schools are widely regarded as more popular) and school are indifferent among all students, the Boston mechanism yields a matching in which every student is weakly better off than in DA [12]. In an empirical investigation into a top Chinese university admission data, Wu and Zhong find that the Boston mechanism might actually perform better in terms of soliciting preference intensity from students, and reward students who have better academic aptitudes but not necessarily higher scores in a single CEE, when students are required to submit their preference list before taking CEE. [13].

2.4 Online Learning and Convergence to Equilibrium

Past theoretical matching research has focused on static aspects of these mechanisms. In reality, however, students and parents learn from their own and others' past experience to decide how to submit their preference list. For example, without the gradual learning process, many parents and students may not understand why truthful reporting should be a best response for them in DA.

One question concerning strategic behaviors in a game is how players arrive at equilibrium. Monderer and Shapley studied a dynamic game process called fictitious play, where each player in each round simply plays the best response assuming the other players strategy is characterized by the empirical distribution of past rounds strategies. They prove that in any every finite weighted potential game, the fictitious play will converge to a Nash equilibrium. [15][18]

Fictitious play in all games will not converge to a Nash equilibrium. Economists have found other algorithmic processes that are guaranteed to converge to a more general class of equilibrium concepts. Foster and Vohra prove that if players have calibrated learning rules, and play best response to their forecasts, the game will converge to a correlated equilibrium, which is a more general concept than Nash equilibrium[17]. Calibrated learning means that when a player predicts a certain mixed strategy of opponents, the asymptotic empirical distribution of the realization of opponents' strategy in these rounds is identical to their prediction. Best responding to calibrated forecasts is known as minimizing the internal regret of players. Hart and Mas-Colell, among other economists, have developed efficient online learning algorithms that are guaranteed to minimize internal regret in repeated games[19]. When these algorithms play against each other in a repeated game, therefore, the empirical distribution of realized strategy profile will converge to a correlated equilibrium.

A weaker notion of regret is called external regret. Given a history of plays in a repeated game, a player has external regret if there is at least one pure strategy which would have generated a higher average utility if the player has been playing it in each round. Algorithms that minimize external regret converge to a coarse correlated equilibrium, which is an even more general notion than correlated equilibrium.

What's the efficiency implication of the instability of IA and PA? How desirable is strategy-proofness in terms of efficiency? How long do students and parents need to learn to play well when a new admission mechanism is implemented? These questions have not been the focus of matching theory research. This paper tries to address these questions with the help of no-regret online learning algorithms.

3 The Model

We can formalize the Chinese college admission problem as:

1. a set of n students $S = \{s_1, \ldots, s_n\}$,
2. a set of m colleges $C = \{c_1, \ldots, c_m\}$,
3. a vector of college quota for a province $q = (q_1, \ldots, q_m)$,
4. a vector of student CEE scores $f = (f_1, \ldots, f_n)$.

We also assume that students and colleges have homogeneous strict preferences over each other. Formally, we assume $c_1 \succ c_2 \succ \cdots \succ c_m$,$s_1 \succ s_2 \succ \cdots \succ s_n$. This is a reasonable assumption in the Chinese context since there is widely accepted ranking of colleges, and this is even more true for tiers of colleges. The CCE scores are supposed to reflect students' academic abilities, which is supposed to be the main criterion in college admission in China.

We can think of the least preferred college as the option of not going to college. Therefore, let $q_m = n$. This is also a reasonable assumption in China as people highly value higher education, partly if not mainly because there is a large difference in job prospect for college and non-college graduates.

The goal of the Chinese education authority here is to find a solution function $\mu : S \to C$ to the Chinese admission problem (S, C, q, f), with the restriction $|\mu^{-1}(c_i)| \leq q_i$, which means that the number of students assigned to a college can not exceed the provincial quota. Note that a technical detail is that some students might not be assigned to any college, which can be reconciled by including a c_0 as no college, with the quota of $q_0 = n - \sum_{i=1}^{m} q_i$. A mechanism M is a function that maps an admission problem to a matching.

In order to learn about the equilibrium behavior under different mechanisms, I use online learning algorithms to model student behaviors when repeatedly playing the college admission game. Students are modelled as Follow the Perturbed Leader (FTPL) algorithms, following the definitions by Kalaia and Vempalab[16]. They have been proven to have the no-external-regret property, so the strategy profile of the algorithmic players will converge to a coarse correlated equilibrium.

Now let's formally describe the FTPL algorithm. Suppose the set of actions is $A = \{1, \ldots, n\}$, before the game starts, FTPL initializes the cumulative payoff vector v such that $\forall i \in A$,v_i is drawn from a geometric distribution of parameter ϵ. Multiply v by the highest possible payoff h. In the period t game:

- FTPL chooses to play the action with the highest current value in v.
- Fixing everyone else's strategy, for each action i, let the payoff of playing i in this round be u_i, and update the cumulative payoff vector by adding u_i to v_i.

4 Results

4.1 Student-Proposing Mechanisms and Sensitivity to Top Choice

In this section, we will explore the property of how a student-proposing mechanism reacts to a change of top choice of a student. I prove that IA satisfies a desirable property that puts importance on a student's top choice, while DA does not. DA satisfies such a property when there is a homogeneous priority of students.

We will first formally define a more general class of college admission problem, and introduce what is a student-proposing mechanism.

Definition 4.1. A general college admission problem is a tuple (S, C, q, P_S, P_C), where S, C, q are defined the same as in the Chinese college admission problem. $P_S = (\succ_{s_1}, \ldots, \succ_{s_n})$ is every student's strict and complete true preference colleges. $P_S = (\succ_{c_1}, \ldots, \succ_{c_m})$ is every college's strict and complete true preference students.

Definition 4.2. Given a college admission problem $p = (S, C, q, P_S, P_C)$, a matching is a function $\mu : S \to C$ s.t. $\forall c_i \in C, |\mu^{-1}(c_i)| \leq q_i$.

Now we will define what a mechanism is.

Definition 4.3. Given a college admission problem $p = (S, C, q, P_S, P_C)$, a student-proposing mechanism is a function $M : (p, P_r) \to U_p$, where $P_r = (\succ_{r_1}, \ldots, \succ_{r_n})$ is the student-reported strict and complete preferences over colleges.

Note that since we are interested in student-proposing mechanisms, students should be able to manipulate their reported preferences under such mechanisms.

Let's first distinguish ex-ante and ex-post stability.

Definition 4.4. A matching μ is *ex-post non-wasteful* if there is not a student s_i and college c_k s.t. $c_k \succ_{r_i} \mu(s_i)$, and $|\mu^{-1}(c_k)| < q_j$.

This property means that if a student reportedly prefers a college with an empty seat to their assigned college, the student should be able to go to their reportedly preferred college. Let's define a similar notion based on students' true preferences.

Definition 4.5. A matching μ is *ex-ante non-wasteful* if there is not a student s_i and college c_k s.t. $c_k \succ_{s_i} \mu(s_i)$, and $|\mu^{-1}(c_k)| < q_j$.

Definition 4.6. A matching μ is *ex-post stable* if it is ex-post non-wasteful, and there is not a student pair s_i, s_j and college c_k s.t. $c_k \succ_{r_i} \mu(s_i)$, $s_i \succ_{c_k} s_j$, and $\mu(s_j) = c_k$.

Definition 4.7. A matching μ is *ex-ante stable* if it is ex-ante non-wasteful, and there is not a student pair s_i, s_j and college c_k s.t. $c_k \succ_{s_i} \mu(s_i)$, $s_i \succ_{c_k} s_j$, and $\mu(s_j) = c_k$.

Now we can define similar properties on student-proposing mechanisms. We define these properties only on ex-post properties of matching since the clearinghouse authority does not observe the true preferences of students.

Definition 4.8. A student-proposing mechanism M is *non-wasteful* if every matching μ in $img(M)$ is ex-post non-wasteful.

Definition 4.9. A student-proposing mechanism M is *stable* if every matching μ in $img(M)$ is ex-post stable.

Remark. *DA is a stable mechanism, while IA is not.*

Now we introduce some new notions on a student-proposing mechanism.

Definition 4.10. Let R_i be the reported ranking over colleges of student i. For a college c, $R_i(c) = |\{c' : c' \succ_{r_i} c\}| + 1$

It's easy to see that there is a natural one-to-one mapping between reporting a ranking and reporting the corresponding preference.

Definition 4.11. Given a student-proposing mechanism M and a set of college admission problem A, *M is sensitive to top choice* in A means:

For all strategy profiles P_R and college admission problem $p \in A$ where there exists s_i, s_j, c_k such that at least one of the following conditions are true:

1. $s_i \succ_{M(p,P_R)(s_j)} s_j$
2. $s_i = s_j$
3. s_j's assigned school is not full.

The alternative strategy profile P'_R where $P'_{R_{-i}} = P_{R_{-i}}$, and P_{R_i} satisfies $R_i(M(p, P_R)(s_j)) = 1$, then $M(p, P'_R)(s_i) = M(p, P_R)(s_j)$. When $s_i \neq s_j$ and the first condition is true, we say s_j is replaceable by s_i. If A equals to the set of all college admission problems, we simply say M is sensitive to top choice.

This property states that if there exists s_i, s_j such that s_j's matched school prefers s_i, then s_i will be admitted if s_i puts the school as their top choice, fixing other students' strategies. This property also states that if a student places their matched school as their top school, they should still be assigned to that school, fixing other students' strategies.

Remark. *IA is sensitive to top choice. Suppose s_i is replaceable by s_j in a matching $IA(p, P_R) = \mu$. Let $\mu(s_j) = c_k$, $s_i \succ_{c_k} s_j$. Since s_j applied to c_k in round $R_j(c_k)$ and is admitted, it must be the case that before round $R_i(c_k)$ c_k is not full. Suppose $R_j(c_k) > 1$, then c_k is not full after round 1, so if s_i applies to c_k in round 1, s_1 will be admitted. Suppose $R_j(c_k) = 1$, it must be that less than q_k students with priority higher than s_i apply to c_k in round 1, otherwise s_j can't be admitted. IA is satisfies the second and third part of sensitivity to top choice by a similar reasoning.*

Note that DA does not satisfy the first condition of sensitivity to top choice in general. In fact, even if s_i is preferred to all currently assigned students in a college c_j, it is not true that if s_i puts c_j as their top choice, s_i will be assigned to c_j. Consider the following counter-example.

Example 4.12. Let $S = \{a, b, c\}, C = \{A, B, C\}$, and the colleges all have the capacity of 1, and have the preferences over students as the following:

- $a \succ_A b \succ_A c$
- $b \succ_B c \succ_B a$
- $c \succ_C a \succ_C b$

Let the above college admission problem be p, and let M be the DA mechanism. First consider the reported preference $P = \{(B \succ_a C \succ_a A), (C \succ_b A \succ_b B), (A \succ_c C \succ_c B)\}$. $M(p, P)(a) = B, M(p, P)(b) = C, M(p, P)(c) = A$. Observe that $a \succ_C b$, so consider the alternative submitted preference where a puts C as their top choice, fixing others' strategies: $P' = ((C \succ_a B \succ_a A), (C \succ_b A \succ_b B), (A \succ_c C \succ_c B))$. Now, $M(p, P')(a) = B, M(p, P)(b) = A, M(p, P)(c) = C$. Since this matching doesn't place a in C, DA is not sensitive to top choice.

Note that DA is sensitive to top choice in the set of college admission problems where colleges have homogeneous preferences over students. The above example shows that s_i can be first assigned to c_j, some other student s_j is rejected as a result, and their subsequent application forced some other student s_k who has higher priority than s_i to apply for c_j, and that means s_i is still rejected. This sequence of events can not happen if colleges have homogeneous preferences over students, since s_j can not kick out any students who have higher priority than s_i in any college.

Clearly, not all mechanisms that satisfies sensitivity to top choice are ex-post stable, as IA is sensitive to top choice but not ex-post stable. On that other hand, not all stable mechanisms are sensitive to top choice, and we have shown a counter-example with DA. This shows that although DA is ex-post stable and strategy-proof, its behavior when individual students change their reported preferences can be more volatile than mechanisms that are sensitive to top choice.

Now we can show the relationship between the Nash equilibrium under mechanisms that are sensitive to top choice and ex-ante stable matching, as students strategically manipulate their reported preferences. Notice that a students' payoff in a matching is characterized by their true preferences P_S, instead of their reported preference P_R.

Theorem 4.13. *Given a student-proposing mechanism M that is sensitive to top choice and a college admission problem p, if P_R is a Nash equilibrium, then the matching $\mu = M(p, P_R)$ is ex-ante stable.*

Proof. Let M, p, P_r be as specified above, we prove by contradiction. Suppose $\mu = M(p, P_R)$ is not ex-ante stable. Then there exists $s_i, s_j \in S$ such that $s_i \succ_{\mu(s_j)} s_j$ and $\mu(s_j) \succ_{s_i} \mu(s_i)$.

Consider the alternative strategy $\succ'_{r_i}$ of student s_i, where the induced ranking R'_{r_i} satisfies $R_{r_i}(\mu(s_j)) = 1$. The alternative strategy is to report $\mu(s_j)$ as their favorite college. Now, consider the strategy profile P'_R, where $P'_{Ri} = \succ'_{r_i}$, and $P'_{Rk} = P_{Rk}$ for any k that is different from i.

Let $M(p, P'_R)(s_i) = \mu'$. Since M is sensitive to top choice, we know that $\mu'(s_i) = \mu(s_j)$.

Therefore, μ must be an ex-ante stable matching. $\square$

Notice that this theorem does not prove the existence of a pure-strategy Nash equilibrium for all sensitive-to-top choice mechanisms. It claims that for sensitive-to-top choice mechanisms that have at least one pure-strategy Nash equilibrium, the equilibrium result must be an ex-ante stable matching.

4.2 The Chinese College Admission Problem and Sensitive-to-Top-Choice Student-Proposing Mechanisms

Coming back to the Chinese case, recall that a Chinese college admission problem is a tuple $p = (S, C, q, f)$. We now assume homogeneous preferences for colleges and students. We denote the two preferences $\succ_s, \succ_c$. The college preference over students is induced by the natural order students scores f. Also, assume that students each has a Bernoulli utility function u_i, and use expected utility to make decisions. Let R_s denote the rank of students since there are only one homogeneous preferences over them. Naturally, $R_s(s_i) = |s_j \in S : s_j \succ_c s_i| + 1$. Without loss of generality, let $R_s(s_i) = i$. Similarly, let R_c denotes the rank of the colleges, and let $R_c(c_i) = i$. In this section, we will assume that CEE scores are perfect signals of a student's ability.

We first show the uniqueness of the stable matching.

Lemma 4.14. *There is a unique ex-ante stable matching in the defined Chinese college admission problem.*

Proof. We know that DA produces a stable matching if students report truthfully, and we just need to show that it is the unique ex-ante stable matching. Suppose there is another ex-ante stable matching λ that is different from the matching produced by DA with truthful reporting, denoted by μ. Let s_k be the first student such that $\mu(s_k) \neq \lambda(s_k)$. Note that it can't be the case that $\lambda(s_k) \succ_s \mu(s_k)$, since if that's the case, there exists $i < k$ s.t. $\mu(s_i) \neq \lambda(s_i)$, which is a contradiction with s_k being the first. So we know that $\mu(s_k) \succ_s \lambda(s_k)$. If $\mu(s_k)$ is not full, then λ is not stable, contradiction. If $\mu(s_k)$ is full, there exists s_j such that $s_k \succ_c s_k$, because $|\{s \in S : s \succ_c s_k\}| \leq q_1 + \cdots + q_{\mu(s_k)}$. Again λ is not stable, contradiction. Therefore, such λ can not exist. $\square$

From now on denote this unique stable matching by μ. Let's formally define the notion of a coarse correlated equilibrium.

Definition 4.15. Given a strategy profile space A, $D \in \Delta(A)$ is a coarse correlated equilibrium if for every player i and every possible pure strategy a_i^* in a's strategy set:

$$E_{a \sim D}[u_i(a)] \geq E_{a \sim D}[u_i(a_i^*, a_{-i})]$$

In a coarse correlated equilibrium, no player would like to opt out of accepting playing action drawn from the equilibrium distribution, and always play one pure strategy. Now we prove the main result of the section.

Theorem 4.16. *Given a sensitive-to-top-choice student-proposing mechanism M in the set of all Chinese college admission problem, every coarse correlated equilibrium $D \in \Delta(\{P_R :$ P_R is a reported preference$\})$ satisfies if $D(P_R) > 0$, then $\forall s_i, M(p, P_R)(s_i) = \mu(s_i)$. This means that every strategy profile with positive support generates the ex-ante stable matching, and implies $E_{P_R \sim D}[u_i(M(p, P_R)(s_i))] = u_i(\mu(s_i))$.*

Proof. We first prove the case for s_1, and we will use induction to prove the rest. Observe that $\mu(s_1) = c_1$. Clearly, since the best college is c_1, $E_{P_R \sim D}[u_1(M(p, P_R)(s_1))] \leq u_1(c_1)$. So we just need to show that it's impossible that $E_{P_R \sim D}[u_1(M(p, P_R)(s_1))] < u_1(c_1)$.

Suppose $E_{P_R \sim D}[u_1(M(p, P_R)(s_1))] < u_1(c_1)$. $\forall P_R$ in the support of D such that $c_1 \succ_c M(p, P_R)(s_1)$. Consider the alternative strategy of s_1 where they report c_1 as their top college. Let the new strategy be $\succ'_{r_1}$, and the induced rank satisfies $R'_{r_1}(c_1) = 1$. We will show that the alternative strategy profile P'_R, where $P'_{R-1} = P_{R-1}$ and $P'_{R1} = \succ'_{r_1}$, will put s_1 in c_1.

Let $\mu' = M(p, P'_R)$. Suppose $c_1 \succ_s \mu'(s_1)$, and that c_1 is not full under μ', then ranking c_1 as top choice will place s_1 in c_1, by the third condition in sensitivity to top choice. If c_1 is full and s_1 is not assigned to c_1 under μ. Then let s_j be a student assigned to c_1 under μ'. We know $s_1 \succ_c s_j \forall j > 1$. Therefore, s_1 can report $\succ'_{r_1}$ to be assigned to c_1 if they originally are not since M is sensitive to top choice. Notice that the same argument also applies to strategy profiles that already assigns s_1 to c_1. Therefore, $E_{P_R \sim D}[u_1(M(p, (P'_{R1}, P_{R-1}))(s_1))] = u_1(c_1) > E_{P_R \sim D}[u_1(M(p, P_R)(s_1))]$, so D is not a coarse correlated equilibrium, contradiction.

Therefore, $\forall P_R$ in the support of D, $c_1 = M(p, P_R)(s_1)$.

Now, assume that this is true for $s_{i-1}, i \geq 2$, we will prove that the same is true for s_i. Suppose $E_{P_R \sim D}[u_i(M(p, P_R)(s_i))] < u_i(\mu(s_i))$, consider the alternative strategy $\succ'_{r_i}$, and the induced rank satisfies $R'_{r_i}(\mu(s_i)) = 1$. We will show that $\forall P_R$ in the support of D, $M(p, (P'_{Ri}, P_{R-i}))(s_i) = \mu(s_i)$. First note that it's impossible that $M(p, P_R)(s_i) \succ_c \mu(s_i)$, $M(p, P_R)$ has assigned better students to schools better than $\mu(s_i)$, according to our induction hypothesis. Therefore, for all P_R in the support of D such that $\mu(s_i) \succ_s M(p, P_R)(s_i)$, suppose $\mu(s_i)$ is not full, then because M is sensitive to top choice, $M(p, (P'_{Ri}, P_{R-i}))(s_i) = \mu(s_i)$. If $\mu(s_i)$ is full, then there must be a student s_j such that $s_i \succ_c s_j$, because $|\{s_k \in S : s_k \succ_c s_i\}| \leq q_1 + \cdots + q_{\mu(s_i)}$, otherwise s_i can not be assigned to $\mu(s_i)$ under DA. s_j is replaceable by s_i at college $\mu(s_i)$, so $M(p, (P'_{Ri}, P_{R-i}))(s_i) = \mu(s_i)$ because M is sensitive to top choice. Finally, for P_R in the support of D such that $M(p, P_R)(s_i) = \mu(s_i)$, $M(p, (P'_{Ri}, P_{R-i}))(s_i) = \mu(s_i)$, because M is sensitive to top choice, so reporting an assigned school as their top choice will result in the same assignment. Therefore, we have proved that by adopting the strategy (P'_{Ri}, P_{R-i}), s_i can get the utility $u_i(\mu(s_i))$ for certainty, so it contradicts with D being a coarse correlated equilibrium. Therefore, it must be that $E_{P_R \sim D}[u_i(M(p, P_R)(s_i))] = u_i(\mu(s_i))$. □

Notice that although DA is not sensitive to top choice in general, it is sensitive to top choice in any Chinese college admission problem. This result shows that any coarse correlated equilibrium under IA and DA in a Chinese college admission problem will generate the fair match with certainty.

4.3 Online Learning Simulations and the Chinese College Admission Problem in the Baseline Model

As we have shown in the previous section, every coarse correlated equilibrium in a sensitive-to-top-choice mechanism will generate the unique ex-ante stable matching in the Chinese College Admission problem. Therefore, when I apply no-regret learning algorithms as strategic preference-reporting students under sensitive-to-top-choice mechanisms in the Chinese college admission problem, they will converge to an a coarse correlated equilibrium. I find that when there is uncertainty involved in the CEE, DA is slightly better than IA in expected efficiency, while IA has lower volatility. The Parallel Acceptance mechanism, which is a more realistic model of the Chinese matching scheme, lies between IA and DA in both expected efficiency. While the variance of PA is close to that of DA, it is in the middle of DA and IA when the population size is large.

As we have discussed in previous sections, the Chinese college admission system has been transformed from a pre-exam IA to a post-exam DA mechanism. For our simulations, we restrict our attention to comparing only the pre-exam version of the mechanisms. First, I believe controlling when preferences are submitted gives us a better picture of comparison of the mechanisms when there is uncertainty. Second, post-exam student-proposing mechanisms will require the introduction of a much richer behavioral strategy space depending on the realized score, thus complicating the learning process. Finally, as we have shown in the previous section, a post-exam mechanism that satisfies sensitivity to top choice will generate the the unique stable matching based on the college preference induced by the realized score, so we can evaluate such post-exam-submission mechanisms just by looking at probability distribution of the random rankings in the CEE.

Let's first define the pre-exam IA, pre-exam DA, and pre-exam PA mechanisms used in the simulations here.

The pre-exam **IA** is defined by the following steps:

- **Step 1:** Students report preferences P_R over colleges.
- **Step 2:** Students take the CEE and obtain scores f.
- **Step 3:** Colleges observe the scores and their preferences over students P_C are induced by the natural order scores.
- **Step 4:** IA is run with q, P_R, P_C.

The pre-exam **DA** is defined by the following steps:

- **Step 1:** Students report preferences P_R over colleges.
- **Step 2:** Students take the CEE and obtain scores f.
- **Step 3:** Colleges observe the scores and their preferences over students P_C are induced by the natural order scores.
- **Step 4:** DA is run with q, P_R, P_C.

The pre-exam **PA** is defined by the following steps:

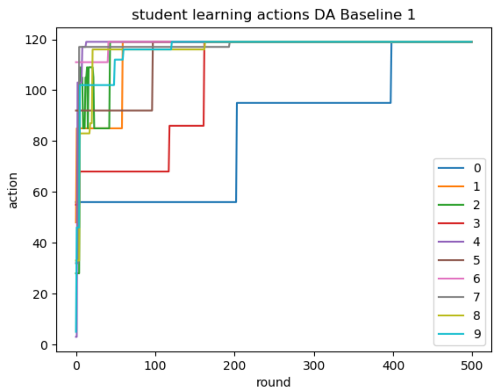
- **Step 1:** Students report preferences P_R over colleges.
- **Step 2:** Students take the CEE and obtain scores f.
- **Step 3:** Colleges observe the scores and their preferences over students P_C are induced by the natural order scores.
- **Step 4:** DA is run with q, P_R, P_C for k rounds.
- **Step 5:** The tentative assignment at the end of k rounds become permanent for students who have been assigned. Subtract the number of assigned students from the capacities of colleges. If there are students without assignment, or if there are students who have exhausted their reported preference list, go to step 6; otherwise, the mechanism terminates.
- **Step 6:** Restart the DA mechanism for the remaining students starting from the next school on their preference list. Run the DA for another k rounds. Go to Step 5.

Notice that pre-exam DA is still a strategy-proof mechanism for students, since for every realized ranking of the students, truthful reporting is always a weakly dominant strategy. Therefore, it is a best response before taking the exam to report truthfully. Intuitively, if a student with low-ability happens to get a high score in the CEE, they have a good chance of being assigned to a better school than their ex-ante fair match, so they had better put the better school in front of their lower fair match in the case. In the pre-exam IA, however, a student with low ability could be risk averse and not put a school much better than their fair match as one of their top schools. Also notice that PA is sensitive to top choice in the Chinese college admission problems, since the previous argument about DA also applies to PA in this case.

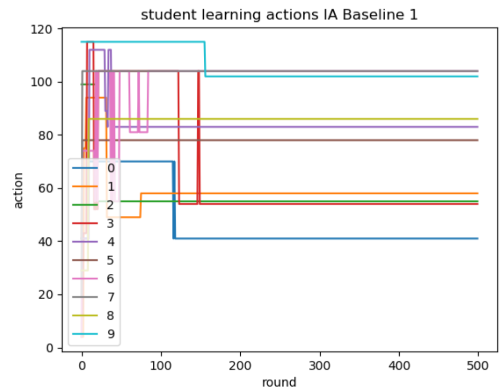
Ideally, if the signals are perfect, every learning result in a sensitive-to-top choice mechanism will end up generating optimal social efficiency. However, since the scores are not perfect, it is unclear which mechanism will perform better in a learning scenario. It is also analytically very hard to pin down the efficiency of mechanisms like pre-exam IA, and the PA mechanism, so the learning algorithms can help us understand the dynamics and effects of these mechanisms.

In the baseline model, I assume that there are 10 students. For each simulation, each student s_i's true academic ability a_i is drawn independently from a normal distribution $N(70, 25)$, and each student s_i's CEE score is drawn from a normal distribution $N(a_i, 25)$. I sort the students with their abilities such that $a_i > a_j$ if and only if $i > j$. I assume that there are 5 colleges, $c_1, \ldots, c_5$, and each student's Bernoulli utility function is such that $u_i(c_j) = j$. The college capacities vector is $q = (2, 2, 2, 2, 2)$. I use the Follow-the-Perturbed-Leader(FTPL) online learning algorithm to model students, and they play the Chinese college admission game for 500 rounds, and in each round they retake the exam and receive a new realized score, while their true academic stays constant throughout one simulation. The FTPL learning parameter ϵ is equal to 0.5. Each student's strategy set is all permutations of reported college rankings. I order these strategies by defining $(1, 2, 3, 4, 5)$ be the first strategy and $(5, 4, 3, 2, 1)$, which is truthful reporting, as the 120th strategy. We calculate the nominal efficiency of any matching μ by $e(\mu) = \sum_{i=1}^{10} a_i u_i(\mu(s_i))$. Optimal nominal efficiency is achieved by the only stable matching when colleges can observe students' true abilities. We calculate the real efficiency of a matching by normalizing its nominal efficiency by optimal nominal efficiency. I choose 2 to be the parameter k in the PA mechanism, which is the number of rounds after which tentative DA assignments become permanent. The baseline dynamic results in one simulation are shown in Figure1,2, and 3. Figure 1 shows what strategies students are playing in each round. Figure 2 shows the evolution of real efficiency. Figure 3 shows the average real efficiency in the last 100 rounds of the simulation.

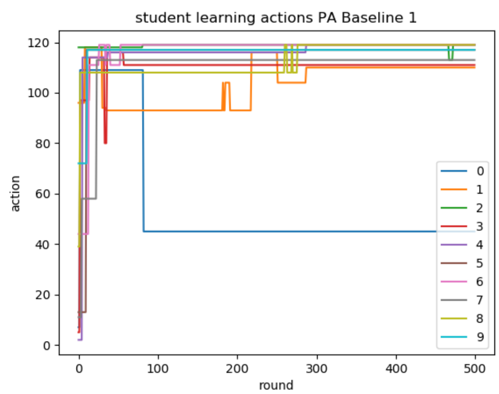
First, note that in pre-exam DA every student's strategy eventually converges to truthful reporting. This is because even the student lowest abilities could end up as one of the top students in a couple rounds, so they learn that putting the top school as their top choice can be beneficial. In pre-exam IA, however, students converge to more complicated strategy



(a) DA



(b) IA

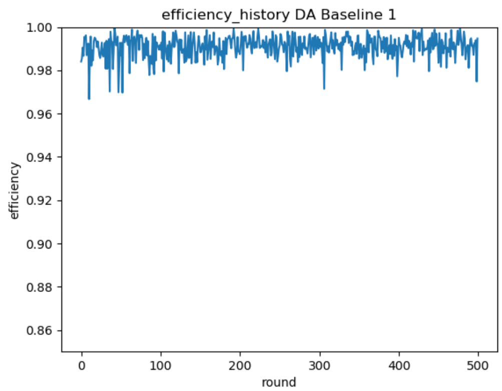


(c) PA

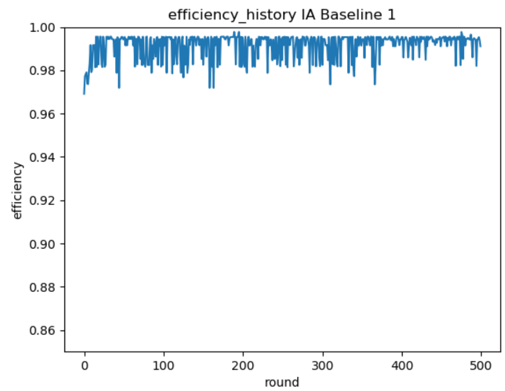
Figure 1: Strategy Convergence in the Baseline Model

profile. Observe that student 6's and 7's reported top school is better than their fair match, and student 8 puts a worse school than their fair match, the best college, as their top choice, because of the competition for the top school. In PA, almost all students put the top school at their top choice except the worst student, but their second choices are different. This shows that compared to IA, the second choice in PA is also very important, and students tend to choose their "match" or "safety" as their second choice, which confirms our intuition about behaviors in PA.

In terms of efficiency, the learning are quick for all three mechanisms. Learning all result in almost optimal efficiency on average in equilibrium, because 10 students drawn from a normal distribution of mean 70 and standard deviation 5 might not be so much different



(a) DA

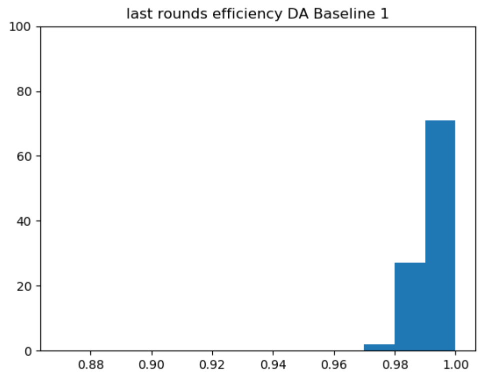


(b) IA

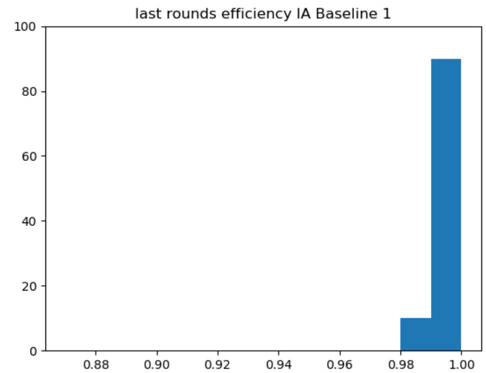
(c) PA

Figure 2: Efficiency Evolution in the Baseline Model

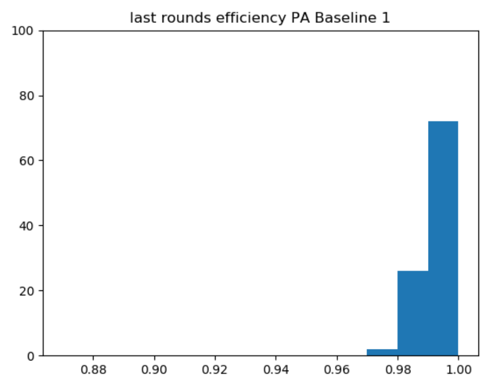
from each other in abilities, which could be the case in reality. Pre-exam IA shows lower variance in efficiency. This is because in the strategy profile where everyone reports their fair match as their top choice, no matter how the scores are realized, they will be assigned to their fair match. The inefficiency of pre-exam IA is a result of more students compete for the top school in the first round, and the loser could go to a much worse school. In addition, if we think of the lowest ranking college in the model as not going to college, it is intuitive that the student with the lowest abilities will still put some college on their preference list, as they can always go to “no college”.



(a) DA



(b) IA



(c) PA

Figure 3: Equilibrium Efficiency in the Baseline Model

4.4 No-Regret Learning Simulations and the Chinese College Admission Problem in Variant Models

In this section, I modify our baseline model and study how the three mechanisms compare in different parameter settings. We explore the mechanism performance under different risk attitudes, number of students, and college capacities. The overall summary stats are documented in Table 1. μ is the average real efficiency in the last a hundred rounds in 10 simulations (6 for the large population variant), and σ is the average standard deviation of the real efficiency in those simulations. In each simulation the academic abilities of students are redrawn and the learning restarts. I find that overall DA scores a little higher in average real efficiency than IA, while IA performance has less variance. PA lies between IA and DA in terms of average efficiency, but resembles DA more than IA in variance.

Table 1: Summary Statistics of Equilibrium Efficiency of Pre-Exam DA, IA, and PA Mechanisms

Experiment	DA μ	IA μ	PA μ	DA σ	IA σ	PA σ
Baseline	0.9916	0.9881	0.9912	0.0051	0.0029	0.0048
Risk Averse	0.9949	0.9929	0.9933	0.0029	0.0021	0.0037
Risk Seeking	0.9869	0.9839	0.9840	0.0079	0.0062	0.0090
20 Students	0.9914	0.9884	0.9896	0.0036	0.0022	0.0033
50 Students	0.9910	0.9874	0.9891	0.0022	0.0013	0.0020
Perfect Signal	1	1	1	0	0	0
Low Variance	0.9964	0.9945	0.9941	0.0027	0.0016	0.0034
High Variance	0.9851	0.9827	0.9839	0.0076	0.0066	0.0082
Pyramid Capacities	0.9913	0.9902	0.9911	0.0056	0.0024	0.0052
Average	0.9921	0.9898	0.9904	0.0043	0.0032	0.0048

Risk Attitudes: In the baseline model, we assign linear utility to the 5 colleges. College 5 has the utility of 5, and College 1 has the utility of 1. We model risk aversion by taking the natural log of linear utilities and add 1. I believe this is a reasonable way to model risk aversion, as by taking the natural log, the difference between the top schools become less significant compared to the difference between the worse schools. In China, it is common to believe that going to a mediocre college is still much better than not having a college education at all. We model risk-seeking behavior by taking the square of the linear utilities. Under this assumption, a student values the top college much more than the second best college, while the worse colleges have less difference in values. Good education resources are concentrated in top Chinese colleges, and graduating from an elite college gives a student a significant advantage in China's super competitive job market. I regard such utility function risk seeking, as a student has more incentives to rank higher a "reach school" instead of a "match school".

As shown in Table 2 the difference in average real efficiency between DA and IA narrows when students are risk averse, while still having the lowest variance. This shift could be due to the fact that if a student fails to be admitted to their top choice in IA, their chance of going to anything better than the worst school is significantly lower than that in DA.

Therefore, students have more incentives to rank their fair match as their top choice, even when their chance of outscoring the student ahead of them is not that low.

When students are risk-seeking, the average real efficiency drops about 0.5% for all mechanisms, and the variance increases overall as well. Two factors contribute to this drop. First, students now have incentives to rank their reach school higher in PA and IA even if their chance of getting in is not as good. This is indeed the case for many Chinese students: they choose to give up a year's admission results hoping to get a better score in next year's CEE. Second, because we square the utilities, the volatility in the top college admission, which now more students want to apply, will be amplified in the overall efficiency. Nevertheless, IA still mitigates this risk-seeking behavior in terms of volatility.

Table 2: 10 Simulations Average Equilibrium Efficiency with Different Risk Attitudes

Experiment	DA μ	IA μ	PA μ	DA σ	IA σ	PA σ
Baseline	0.9916	0.9881	0.9912	0.0051	0.0029	0.0048
Risk Averse	0.9949	0.9929	0.9933	0.0029	0.0021	0.0037
Risk Seeking	0.9869	0.9839	0.9840	0.0079	0.0062	0.0090

Number of Students: In the baseline model, I look at simulations where there are 10 students. Coordination among more students can become increasingly difficult, as the potential strategy profile grows exponentially. When we have more students, it might be reasonable to hypothesize that DA becomes superior, since the the dominant strategy is simple, while IA and PA will require more sophisticated strategizing. This is indeed one of the main concerns with IA. Therefore, we increase and number of students to 20 and 50 in two variant simulations. We respectively increase the capacities of the colleges to 4 and 10.

The simulation results show a consistent pattern between the mechanisms as the baseline model. The reason why IA still exhibits low variance is that the IA dynamic might not be complicated enough for the learning algorithms to quickly learn to coordinate. In reality, however, parents and students might not have a rich knowledge of the matching history, and could very well misbehave and choose the suboptimal strategy, especially given other parents could make the same "mistakes" as well. It is worth noticing that as the number of students increases, the average real efficiency drops slightly. For DA this might be due to situations such as at least some worse student at the margin of a better school outscoring a better student becomes very likely. For IA, this might be due to students at the margin of a better college willing to take risks. Finally, the standard deviation of the three mechanisms all drop, maybe due to the law of large number effect narrowing the probable range of realized real efficiency.

It's worth noticing that although PA shows DA-level variance when population size is small in other variants, its variance is strictly between IA and DA when the population size is large.

Table 3: 5 Simulations Average Equilibrium Efficiency with Large Student Population Sizes

Experiment	DA μ	IA μ	PA μ	DA σ	IA σ	PA σ
Baseline	0.9916	0.9881	0.9912	0.0051	0.0029	0.0048
20 Students	0.9914	0.9884	0.9896	0.0036	0.0022	0.0033
50 Stuents	0.9910	0.9874	0.9891	0.0022	0.0013	0.0020

Exam Variance: In the baseline model, each student's CEE score in each round is drawn independently from $N(a_i, 25)$, where a_i is each student's true academic aptitude. To give more concreteness to the randomness in the score model, a student who is 1 standard deviation, which is 5 expected points, behind a better student, has about 24% chance of outscoring the better student. We explore here the cases when the exam variance is 0, 9, and 100, which corresponds to the above surpassing probability of 0%, 12%, 36%.

Suppose the variance is 0, which is when the CEE score is a perfect signal of a student's academic abilities, the problem reduces to the one discussed the previous section without uncertainty. As shown in the table below, the simulation does confirms our theoretical finding as the real efficiency is simply always 1 for all three mechanisms, as indicated by Theorem 4.16.

As I increase the variance, the overall efficiency drops for all three mechanisms and the variance all increases, which is reasonable, as the realized ranking of the students resemble less the true ranking. Suppose the variance of the exam is sufficiently large, the efficiency distribution of the mechanisms could drop to close to a complete random assignment. In DA, as students learn to report truthfully, the final assignment all depends on the realized ranking, so the more randomness there is in the ranking the less efficient the assignment will be. For IA and PA, higher variance of the exam incentivizes students to apply for their "reach" schools, especially for those whose fair assignments are the worse colleges, as losing the bet will only result in a slightly worse outcome compared to those whose fair match is already good.

Table 4: 10 Simulations Average Equilibrium Efficiency with Different Exam Variances

Experiment	DA μ	IA μ	PA μ	DA σ	IA σ	PA σ
Baseline	0.9916	0.9881	0.9912	0.0051	0.0029	0.0048
Perfect Signal	1	1	1	0	0	0
Low Variance	0.9964	0.9945	0.9941	0.0027	0.0016	0.0034
High Variance	0.9851	0.9827	0.9839	0.0076	0.0066	0.0082

College Capacities: In the baseline model, we assign equal number of seats to each college. In reality, many Chinese colleges do accept approximately the same number of students, as the majority of colleges are public colleges. However, there exists a widely acknowledged hierarchy of tiers of Chinese colleges. Tsinghua University and Peking University are usually recognized as the top 2 colleges in China, and 39 colleges in "Project 985" are considered the elite colleges. The 112 colleges in "Project 211" are considered the next tier of good colleges. All these colleges are officially categorized as tier 1 college, among other colleges, and there are tier 2 colleges as well. Therefore, we explore a variant where we assign $q = (3, 3, 2, 1, 1)$.

As shown in Table 5, the variant statistics are not a lot different from the baseline model. This suggests that college capacities don't influence the efficiency of mechanisms a lot. In reality, the limited capacities at good colleges could make students value seats at these colleges even more, as going there can be a stronger signal of a student's ability.

The variant models show a coherent landscape of the three mechanisms overall: DA is slightly superior in expected efficiency while IA is more stable in performance, and PA lies

Table 5: 10 Simulations Average Equilibrium Efficiency with Different College Capacities

Experiment	DA μ	IA μ	PA μ	DA σ	IA σ	PA σ
Baseline	0.9916	0.9881	0.9912	0.0051	0.0029	0.0048
Pyramid Capacities	0.9913	0.9902	0.9911	0.0056	0.0024	0.0052

in between. Since the random process of getting a CEE scores is identical for the mechanisms in a given variant, the difference in performance is due to what equilibrium strategies the students learn to play. Because of the special importance of the first choice in IA, if students learn to put their fair match as their top choice, no matter what the realized ranking of students is, IA will generate the fair match. This is not true for DA, as students learn to truthfully report. However, under IA if a student is willing to apply to a college that is better than their fair match and take the risk of potentially getting an assignment that is much worse, because they are not far behind in abilities, such strategy profile contributes to the inefficiency in IA. On the other hand, in this way IA allows students to put their risk attitude and preference intensity into consideration in the application process. Therefore, IA serves like an insurance policy for students who are risk-averse, and also allows space of strategizing for risk-seekers.

PA, as a combination of IA and DA, permits a combination of conservative and risky strategies. For example, in our baseline PA model where after 2 rounds of DA the tentative assignment becomes permanent, a student can put a “reach” school as their top choice, and then put their fair match or a safety school as their second choice. PA offers a combination of an insurance policy and an opportunity some level of truthful reporting.

5 Conclusion

As millions of Chinese high school graduates are placed into college through the College Entrance Exam, or Gaokao, and a matching process, how the matching system is administered has profound impact on individual student’s future as well as education resource allocation efficiency in the Chinese society. The matching system has since 2000 transformed from a pre-exam IA style to a post-exam DA style mechanism, attempting to reduce the ex-post blocking pairs in the matching result. However, the efficiency of the mechanisms are unclear.

This paper adopts the framework of theoretical matching theory to explore various matching mechanisms. We show that in a general college admission problem setting, under any mechanism that satisfies the sensitivity to top choice property, a Nash equilibrium outcome will always generate an ex-ante fair match. Then we show that when the CEE scores are perfect signals of students' underlying academic abilities, which means there is no uncertainty involved in colleges' preferences, IA, DA, and PA mechanisms will generate the ex-ante fair match with certainty in any coarse correlated equilibrium.

We turn to no-regret online learning algorithms to have an understanding of equilibrium outcomes when there is uncertainty in the realized CEE scores. We model students as online learning algorithms that have constant academic abilities, but play the preference-submission game each round with new realized CEE scores. In the baseline and the variant models, we discover that DA slightly outperforms IA in terms of mean efficiency in equilibrium outcomes, while IA consistently outperforms DA in terms of achieving a lower variance. PA lies in the middle of the two mechanisms in both efficiency mean and variance.

In reality however, how much rationality we can assume for parents and students is not clear. There is definitely a learning process where parents and students study previous year's admission results, but this learning process is not exactly like taking the CEE and playing the preference submission game for a large number of years. In addition, coordination between hundreds of thousands of parents and students in a province can be very different from a coarse correlated equilibrium, as no one is certain of everyone else's strategy, not even the distribution of others' strategies given what oneself is planning to play. Still, our theory and simulations shed light on some basic properties of the different mechanisms.

The policy implication is that there is a trade-off between expected efficiency and the volatility of the efficiency. A social planner can choose from a class of PA mechanisms in order to optimize their objective. In addition, all these mechanisms achieve comparable high levels of efficiency. Therefore, if there is a high switching cost of changing an established system, the change might not be worth the administrative cost plus the time people need to adapt to play well in the new system. Further studies can be done on how students with different risk attitudes, different levels of sophistication at strategizing, different access to information regarding others' strategies and past admission records, etc. compare with each other in admission results, as these questions touch on a deeper level of fairness, which is essential for the success of any college admission mechanism.

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