Residual Return Momentum: Behavior and Performance

Rajiv Chopra

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Abstract

Traditional momentum strategies have been built using total returns to build winner and loser portfolios, but recent literature has revealed that this approach causes investors to inadvertently take exposure to other systematic risk factors. This can often lead to catastrophic downturns around panic periods, and misinformed diversification choices for investment portfolios. Instead of using total returns, I use residual returns extracted from factor regression which control for the stock’s systematic risk exposures. I then build momentum portfolios by ranking stocks on their residual returns. I do sensitivity analysis on residual return momentum to its parameters and confirm that the strategy’s comparative statics are in line with intuition, and lend themselves to sensible trading practices. I then explore the behavior and performance of a particular parameter set, revealing that residual return momentum consistently produces alpha while taking limited exposure in market beta, value, size, and total return momentum.
1 Introduction

1.1 Empirical Asset Pricing

Empirical asset pricing models were born into the cannon of financial literature as early as the 1960’s, with the introduction of the Capital Asset Pricing Model (CAPM) as the first major breakthrough in this regard (Sharpe, 1964). This model, and the extensions which followed, all followed the underlying principal that any financial security can be priced based on its exposure to a set of market risk factors. These models believe that there is a risk premium (return) associated with each risk factor – so by holding a security that has exposure to any risk factor, one should expect to be compensated accordingly.

In the case of the CAPM, the simplest model, the only risk factor is the market itself:

$$R_{i,t} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_{f})t + \epsilon_{i,t}$$

To be exact, the CAPM states that a stock’s expected excess return should be explained entirely by its correlation the market risk premium, or the excess return of the market over the risk free rate (Sharpe, 1964). This exposure is referred to as a firm’s market beta. Recall that each factor represents an exposure to risk – since the risk-free rate is by definition not a risk, I should only be compensated for the excess risk I take from the market.

As a first attempt to price securities, the CAPM did an impressive job given its nascent nature, with market beta explaining ~85% of diversified portfolio returns (Womack, 2003). However much of the model’s appeal is in its simplicity - in the theoretical framework of a static market where every asset is perfectly liquid and investors are perfectly rational, the CAPM did in fact prove to be very elegant in its exact correctness (Sharpe, 1964). Unfortunately, as one might expect, markets are far less well-behaved than the CAPM
assumes, and so research continued in the pursuit of additional factors to explain additional components of security returns.

Many years after the CAPM was introduced, two professors from University of Chicago, Eugene Fama and Kenneth French, made the next major breakthrough in the empirical asset pricing space by introducing what is now commonly referred to as the “Fama French 3-Factor Model”:

\[
R_{i,t} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_f)_t + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \epsilon_{i,t}
\]

This model introduces two additional factors to the CAPM, the “HML” factor and “SMB” factor. These factors are meant to represent firm value and firm size as additional risk factors, respectively. Recalling the underlying assumption to any factor model, this models claim was that one should be compensated for holding a stock not only due to its exposure to the market, but also because of the additional risk associated with its value and size. In the case of value: HML stands for “high minus low” and refers to the book value-to-market value ratio (B:M) of a particular stock. It was found that the equity of firms with high B:M (often referred to as value stocks), when holding market beta and firm size constant, tend to outperform those with low B:M (growth stocks) in the long run. In the case of firm size: SMB stands for “small minus big” and refers to the market capitalization of a particular stock. Here they found that smaller firms tend to outperform larger firms in the long run when one controls for market beta and B:M (Fama/French, 1992).

Fama and French proved the significance of these factors by building diversified portfolios to represent each factor, and then regressing various portfolio returns on the returns of these factor portfolios. The HML and SMB factors are constructed as specified on the Fama French website: they use 6 value-weighted portfolios formed on size and book-to-market. SMB
is the average return on the three small minus three big portfolios. HML is the average return on two value minus two growth portfolios:

\[
    \text{SMB} = 1/3(SVal + SNeut + SGrowth) - 1/3(BVal + BNeut + BGrowth)
\]

\[
    \text{HML} = 1/2(SVal + BVal) - 1/2(SGrowth + BGrowth)
\]

With this technique they were able to explain ~95% of well diversified portfolios’ returns, and financial researchers across the board adopted their model almost immediately as the new performance benchmark (Womack, 2002). This adoption came in large part because of the empirical support for their findings, but also because there was also a strong intuition behind why their factors made sense. Recall that these models explain returns as compensation for systematic risk; therefore stocks with higher returns must exhibit some sort of additional explained risk exposure. In the case of SMB, it’s quite intuitive to imagine that small firms would be slightly riskier as they have less ability to absorb shocks which large firms might be able to easily internalize. In the case of HML, it’s slightly less simple but basically boils down to the fact that value firms are those whose market value has dropped far below its book value due to doubt regarding future earnings, especially since these firms tend to be those with lower earnings growth rates (Womack, 2002). In both cases, there is a clear risk exposure which is not captured by market beta, but exists even when all other factors are held equal.

1.2 Momentum

Around this same time, a potential fourth factor was being explored which sparked quite a bit of controversy in the empirical asset pricing space, which people colloquially referred to as “momentum” (Carhart, 1997). This factor was founded on the simple notion that recent “winner” stocks will continue to outperform recent “loser” stocks, where winners (losers) are defined to be the stocks with the highest (lowest) cumulative returns over recent months. The specific mechanics of return momentum vary along two main dimensions: the
look-back window (how long of a window you use to build the cumulative return factor), and the holding period (how long you then hold your positions). Intuitively, the look-back window represents to how valuable you believe past information to be: do I think returns 24 months ago are important to what returns will be in the future, or do I want a shorter window — maybe more like 12 or 6 months? The holding window represents how long you believe the lifespan of the momentum to be: will this stock continue to outperform for 1 day? 1 week? 1 month? 1 year? Although it varies from one study to the next, typical momentum portfolios have a look-back of 6-12 months and a holding period of 6-12 months (Jegadeesh, 2001).

In a purely empirical sense, this factor has found overwhelming support in the cannon of financial literature, stretching even into other asset classes and markets across the world (Asness, 2013). However, from an intuitive standpoint, this factor has been a bit more controversial. Cliff Asness, one of the major proponents of momentum investing, even admits that the logic behind the existence of momentum is almost impossible to determine. He says that momentum can be explained as a result of either investor over-reaction or investor under-reaction, admitting that completely opposite explanations are equally plausible. And at the end the day, one does not need to understand the why there is momentum in asset prices in order to take advantage of it as an investor. That said: this lack of understanding and consensus over why momentum exists has sometimes, albeit rarely, produced pretty devastating results, where momentum investors are left trying to figure out what happened to all of their money (Daniel, 2013).

For example: momentum strategies have been found to be fairly steady over long periods of time, with rare but severe periods of consistent negative returns. (Daniel, 2013) For a long time, researchers had trouble understanding why this was happening, only really noting
that it happened around panic periods. Their intuition was that in the event of a panic period, investors will scramble to liquidate the assets they can – since momentum stocks tend to be the most crowded and liquid stocks, these are the ones which take the largest fall when everyone rushes to sell them off at the same time (Asness, 2013). But this was largely speculation, and researchers only felt the need to understand that momentum loaded positively on liquidity risk, without being concerned about why. After all, it’s not necessary to understand the logic behind a risk premium in order to trade it.

Then in a recent paper by Kent Daniel and Tobias Moskowitz, a finding was made that opened up the potential for a new way to understand momentum. This paper, entitled “Momentum Crashes,” decided to decompose momentum portfolios around panic states in the hopes of better understanding this phenomena for momentum portfolio. Their findings were very interesting, but fairly intuitive: in a bear market, your momentum portfolio returns are driven heavily by their beta loading. Specifically: in the period that a market is declining, your momentum portfolio will invest heavily on low/negative beta stocks (IE, ones whose correlation to the market is low or negative), as those have been performing relatively well in the period of market decline. At the same time, it will sell of high beta stocks which have been suffering in the recent period. Naturally, when the market recovers, your momentum portfolio suffers losses due to your loading – what was interesting though was that the loss in the recovery far outweighs the gains during the downturn. The research found that this is due to an asymmetry between the “winners” and “losers” momentum portfolios: the stocks in your loser portfolio (high beta) have an especially high market premium attached to them during recovery, much more than the premium attached to the winner stocks (low/negative beta) during the decline period. They offered the potential explanation that low beta stocks offer
an additional “insurance” premium which dampens their returns during busts relative to what high beta stocks receive during booms.

Without getting too bogged down in the details of their findings, the important thing to note is the following: this research revealed a major flaw in momentum portfolio construction, namely that total returns are not controlled in any way for their exposure to other factors (in this case, market beta). Recall that when we defined SMB and HML, both factors were constructed holding all other factors constant. Momentum portfolios are fascinating in that they have always been constructed, and largely been successful, using unconditioned returns. But as Daniel reveals, there is a hidden danger in that the returns of these momentum portfolios are difficult to disentangle from the returns to other factors. As a momentum investor you should therefore be cognizant of the fact that a portion of your returns are actually due to the portfolio loading on the other risk factors. Rather than just taking on momentum risk, you are actually taking on all factor risk.

2 Residual Return Momentum

It is by this line of thinking that Blitz, Huij, and Martens created a simple but intuitive approach to build momentum portfolios on returns that are controlled for their various factor exposures. Mechanically, they use the residual of a factor regression. The residual of a regression represents the portion of the dependent variable which cannot be explained by the independent variables. In the case of an asset pricing factor model, the dependent variable is a security’s return, and the independent variables are the risk factors. By using the Fama French 3-factor model, they extract a residual defined exactly to be the portion of a firm’s returns that is not explained by its market beta, its size, or its value. They then build a
momentum strategy out of these residual returns, and find that this momentum portfolio has a significantly lower factor exposure than a total return momentum portfolio (not surprising, given how it is constructed).

While their research is very useful in its suggestion to use residual returns, the core focus of their research was on portfolio performance rather than behavior, and limited to one particular set of parameter choices - it leaves an enormous amount of room for continued exploration. Namely, if we think about this strategy, it has three major components which we can vary and analyze which their research chose not to do:

1) How do you construct the residual returns?

2) How do you turn those residual returns into a momentum factor?

3) How do you construct a portfolio using that factor?

In the research by Blitz and co., they used a single approach throughout. For constructing their residual returns, they used only the Fama-French 3 Factor model. For building their momentum factor, they normalize their residuals using the standard deviation calculated over the past 36 months, and then cumulate those normalized residual returns from months (t-12) to (t-1). Cumulating returns to month (t-1) rather than month t is standard in momentum portfolio construction because of observed short term reversion (Gutierrez, 2007). Finally, to build their portfolio they use a holding period of 1 month.

While their research was focused on the performance of this particular portfolio, my goal is to explore how residual return momentum changes behavior based on adjustments to these three component of the strategy: the residual extraction, the factor creation, and the portfolio construction. Recall that the challenge in understanding total return momentum is that it is entangled with the other factors. When we are looking at residual returns, we are

8 of 36
looking exclusively at the idiosyncratic, firm-specific portion of returns – the hope is that by understanding residual momentum we can construct a strategy which more accurately predicts periods of future returns without also loading heavily on factor risk.

3 Data and Methodology

For my stock data I use monthly data from the CRSP database – specifically, all domestic, primary stocks listed on the New York (NYSE), Nasdaq, and American (AMEX) stock exchanges. I limit my study to US equity between January 1926 and December 2013. All of my factor data comes from the Fama French website.

3.1 Residual Extraction

The first step of our process is extracting the residual returns from our factor regressions. As I said, the goal here is to see how the portfolio’s behavior changes based on how the residuals are formed which are used to construct it. I use 3 different factor models to construct the residual returns:

3-Factor Residual Extraction Model

\[ R_{i,t} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_{f})_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \epsilon_{i,t} \]

This is just the Fama French model, which is what was used in the Blitz paper – although nothing differs in the residual extraction, I do vary the parameters of the factor and portfolio constructions to better understand our strategy’s behavior.

4-Factor Residual Extraction Model

\[ R_{i,t} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_{f})_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{MOM}MOM_t + \epsilon_{i,t} \]
For this next model, I actually add the total return value-weighted momentum portfolio as a 4th factor. It may seem a bit unintuitive, but the explanation is as follows: total return momentum, recall, is a factor constructed from the universe of stocks and is meant to represent market-wide momentum. When I regress stock \( i \)'s returns on the total return momentum portfolio, I am only extracting the portion of stock \( i \)'s return which is correlated with common-factor (market-wide) momentum, and leaving the portion of stock \( i \)'s return which exhibits firm-specific momentum. The specific dataset used for our momentum factor is the monthly return series to the value-weighted momentum portfolio on the Fama French website, going long the top 10% portfolio and short the bottom 10% portfolio.

5-Factor Residual Extraction Model

\[
R_{i,t} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_{f}) + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{MOM}MOM_t + \beta_{ind}R_{ind(i),t} + \epsilon_{i,t}
\]

In this final model, I regress stock \( i \)'s returns on our 4 factors plus an additional industry factor. This industry factor is simply the value-weighted return to equity in the industry which stock \( i \) belongs. If there is still momentum in the residual of this regression, it means that there is still some unexplained firm-specific momentum which exists above and beyond all of our factors and above industry exposure. For the industry factor, I use the monthly return series to the value-weighted industry portfolios separated into 17 industry buckets. We map our stocks into the appropriate industry using the SIC-industry mapping provided by Fama and French on their website.

For the regression parameters I use a rolling regression with a 3-year window calculating the residual return at period \( t \) using coefficients estimated from time \((t-36)\) to \((t)\). Any period with missing return data can’t be estimated, and because the factors all have complete data I never need to use in-sample estimation for the residuals.
3.2 Factor Formation

Once the residuals are calculated, we need to transform them into a residual return momentum factor, which we will use to construct a residual return momentum portfolio. For the factor construction, recall we can either normalize or leave residuals untouched, and we can vary the cumulative return look-back window. In my analysis, I’ve used both normalized (by the same method as in the Blitz paper) and raw (untouched) residual returns. For the look-back window, which is how long we accumulate our residual returns, I’ve used 3, 6, and 12 months; in all instances I’ve dropped the most recent month as is common practice. In other words, for time period \( t \) and look-back window \( J \) our raw/normalized factor is constructed as follows:

\[
Raw_t = \sum_{k=t-J}^{k=t-1} R_{resid,k} \\
Norm_t = \sum_{k=t-J}^{k=t-1} \frac{R_{resid,k}}{\sigma_{R_{resid,k-36}}}
\]

\( \sigma_{R_{resid,k-36}} = \text{Std Dev calculated from } k, \ldots, k-36 \)

Before moving on, let’s take a step back to think about the concept behind each component in this factor construction. Normalization is typically done to bring data measured in different units into comparable units. Data can be incomparable across time and incomparable across stocks (cross-sectionally). Typically, incomparability across time is due to things like inflation (IE, a change in price of $1 60 years ago is much more significant than a change in price of $1 today). Incomparability across stocks can come from any number of sources, but it’s primarily due to the magnitude of data for one member of the cross-section being substantially different from the magnitude of another member. In the case of residual returns, there’s no reason why data would be incomparable across time, and by normalizing across time we may even run the risk of underemphasizing certain periods where momentum may be strongest for a particular stock. There may be an argument for why data may be
incomparable across stocks, though; namely, if we believe that a stock’s firm specific momentum component is based not entirely on the magnitude of returns over the past period, but rather the extent to which its returns have deviated from its average. If it’s the case that normalized residual momentum performs better than the non-normalized version, that may be an indication that firm-specific momentum has to do not only with how an individual stock performs relative to other stocks, but also relative to its own past performance. My prior would suggest that non-normalized returns would make more sense, for a number of reasons: A) normalization will make you more likely to invest in a stock with low return magnitude, which even if it’s been outperforming relative to its own recent performance, the future returns are still likely to small in magnitude relative to other stocks which you could have purchased. B) Total return momentum works on non-normalized returns, giving us no reason to believe the residual returns should be adjusted. That said, confirming or refuting this being the case will still reveal something important about firm specific momentum: namely, is relative outperformance compared to past self or present cross-section more important?

The look-back window (J), recall, is the month (t-J) which we begin our cumulative return sum in order to build our momentum factor at time t. Recall that we drop the most recent month (t-1) as is common practice. The look-back window basically represents how long ago we think there exists valuable firm-specific information in predicting future momentum. Quite intuitively, the longer we think the information remains relevant, the longer we want to make our look-back window. Another way of interpreting it is how fast we think the factor “moves”. Roughly speaking, a factor’s movement corresponds to how much it fluctuates from one time period to the next. If we use a short look-back window, we’re assuming that momentum moves quickly and our momentum factor will change direction rapidly; a longer look-back window will produce a flatter and slower momentum signal, as
each new month is a smaller component of the total sum and therefore doesn't cause major
direction changes month to month. Note that it is also possible to create a more sophisticated
weighted or kernel-fitted cumulative return sum, but that is not the purpose of my research
and doesn’t add any additional insight into how fast the momentum factor moves. For my
purposes, just varying the look-back window will provide ample information on the speed of
firm-specific momentum.

3.3 Portfolio Construction

With the factor created, the final step is to translate the factor into actual portfolio
positions. In terms of methodology, it’s exactly like any other momentum strategy: at the end
of each month, we create a portfolio containing the top 10% of stocks ("winners"), and a
portfolio containing the bottom 10% of stocks ("losers"). We go long the portfolio of winners
and short the portfolio of losers, creating a zero-cost momentum portfolio. In terms of specific
mechanics, there are two dimensions which we can vary: weighting and holding period.
Weighting refers to how I distribute my exposure across the stocks in my portfolio in each
time period. The two main approaches are A) equal weighting, where we put the same weight
into each stock, and B) value weighting, where we weight according to the market
capitalizations of the stocks in the portfolio. Mechanically, for each period, weights are
constructed as follows:

\[
Equal : w_{pf,t}(i) = \frac{1}{|Stocks_{pf,t}|}
\]

\[
Value : w_{pf,t}(i) = \frac{MktCap_i}{\sum_{s \in pf} MktCap_{s,t}}
\]

Note that value-weighted portfolio self-rebalances throughout its holding period, because any
variation in your weighting due to price changes will be exactly reflected by changes in the
various market capitalizations of each stock. Equal weighting, on the other hand, must be rebalanced every month regardless of how long your overall holding period is, as price changes will cause you to be overexposed to stocks whose prices rose above the cross-sectional mean, and underexposed to those that didn’t.

Holding period refers to the length of time which I hold a position from the date where my factor indicates me to do so. My overall portfolio, regardless of how I define my holding period, rebalances every month – but with $H>1$, my overall portfolio is technically composed of $H$ different overlapping portfolios, 1 of which rebalances every month. By this methodology, $1/H$ of my total position are rebalanced monthly, and the position of my overall portfolio is just the average position across my $H$ overlapping portfolios. Note that in the case of equal weighting, we technically rebalance every overlapping portfolio each month in terms of dollar amounts because any price change will also cause a weight change – but as far as our weights ($\%$ of portfolio composed of stock $j$) is concerned the above logic is still exact. In the context of my portfolio construction, I create portfolios with holding periods of 1, 3, 6, and 12 months.

Again let’s take a step back and understand the implications of each of these design choices. First, as it relates to weighting: typically, equal-weighted momentum portfolios tend to outperform value-weighted momentum portfolios. There are three reasons associated with why this might happen: first, equal-weighted strategies have been found to put higher weight on small stocks (mechanically so) and value stocks (Plyakha et al., 2012). In the case of residual momentum, it’s not guaranteed that this effect will still exist on our portfolio returns, as our residuals were built controlling for these factors and so the momentum factor built from these residuals should exclude any portion of momentum which is driven by either factor. The second reason equal-weight outperforms is because mechanically, it takes a contrarian view:
in order to stay rebalanced, it sells stocks who’ve outperformed (price increased more than
other stocks in the portfolio), and it buys those who have underperformed. This is relatively
counterintuitive as it relates to momentum, which is essentially trying to do the opposite –
and in fact, value-weighted momentum often outperforms equal-weighted momentum when
using total returns. It’s hard to form a prior on how this contrarian mechanic will affect our
residual return portfolio, because recent absolute losers may in fact be residual winners if they
outperform their benchmark – but one still wouldn’t necessarily expect this to be the cause.
A final reason this may occur is because Plyakha et al. found that the idiosyncratic volatility
of assets in a portfolio cause an increasing relation to the difference between the equal and
value-weighted version of that portfolio. If it’s the case that our equal weighted portfolio
outperforms our value-weighted portfolio, this is what would make the most sense.

Our next parameter, holding period, may seem to be the same thing as the look-back
window but actually captures a different belief. The look-back window, recall, is an indication
of how fast we believe our signal moves (how much we allow it to vary from month to month).
The holding period, on the other hand, is an indication of how persistent we believe returns
to be for those stocks which our factor tells us to buy. Is the residual momentum factor able
to predict returns only for next month, or can my momentum factor predict returns further
into the future? By definition, this relies on returns being persistent, which has found
overwhelming support in the literature (Jegadeesh, 1993); after all if I make my position at
time t and hold it until time t + H, then I’d obviously hope that returns are positive
throughout that entire holding period (IE, they’re persistent). The success of various holding
periods reveals the degree to which residual returns can predict periods of persistent future
returns.
4 Sensitivity Analysis

Note that Figure 1 summarizes section 3 in a concise table. In total, I have \((3 \times 3 \times 2 \times 2 \times 4) = 144\) parameter combinations (meaning 144 different portfolios). However the purpose of sensitivity testing here is not to just pick the best portfolio of those 144 and move on. Sensitivity testing here is more of a sanity check that residual returns behave in a way that makes sense, and that the portfolios reveal some pattern in how returns change based on changes in the parameters. If I am confident that these portfolios behave in a way that makes sense given my intuition on the implications of parameter choices, then I will pick a single set of parameters and explore the behavior of that portfolio in greater depth. If there is no consistency in the behavior of these portfolios, then one should be skeptical to incorporate residual return momentum in their own portfolio.

It might seem as if I’m beating a dead horse with the emphasis on the portfolio behavior “making sense” or “matching intuition,” but the nature of this strategy is that it’s one that’s very easy to data-mine. While there could be a very specific set of parameters that create extraordinary returns, one should be very skeptical of those results if there is no sensitivity analysis accompanying them. And if quantitative finance has taught is anything, it’s that putting too much faith into something which seems good in theory may have disastrous results in practice if it’s not well understood before being implemented.

Figures 1-6 summarize our sensitivity analysis by showing the average performance (as measured by annualized Sharpe) and exposure of all portfolios across each dimension. So in Figure 2, the column “3” shows us the average performance of all portfolios build from residuals extracted from the 3-factor model (allowing all other parameters to vary).
4.1 Residual Extraction

For our residual extraction, the portfolios appear to lose a small amount of Sharpe with the addition of each factor, however the alpha does increase. For this to be the case, the correlation to other factors of our portfolio must be decreasing – and we can see that this is the case. So although the risk adjusted returns suffer slightly, you end up loading on smaller amounts of systematic factor risk. Mechanically, this is somewhat intuitive – if our residuals represent non-systematic (idiosyncratic risk), then the addition of each new factor will push those residuals towards zero by removing the portion of the residual explained by each additional factor. So although our 5-factor model has a slightly lower average Sharpe, and recall that these are averages, the diversification benefit of lower factor loadings more than compensates. Recall that this sensitivity analysis is largely a sanity check – if I really were interested in dataminining the best portfolio I could easily run statistical test to determine the significance in difference between the average Sharpes and beta exposures. But again, that’s not the purpose of this analysis.

4.2 Factor Construction

Moving onto the factor construction, recall that we have two choices: whether or not to normalize our residuals, and the look-back window for how many months of cumulative returns we want to roll up. Normalization (0 meaning non-normalized, 1 meaning normalized) appears to have little impact on the averages of our portfolios, with regards to both performance and behavior. Our normalized portfolios do appear to load slightly less on mom, but at the same time load slightly more on SMB – without a strong prior on why this would be the case, and with such a small difference in the factor loadings, it doesn’t seem worth it to explore in greater depth. Given that our prior was that returns didn’t need to be
normalized, and the similarity of our portfolios regardless of normalization, it seems best to leave our residual returns un-normalized.

The choice of look-back, on the other hand, has a drastic impact on the performance of the portfolio. With average Sharpes of .1, .28, and .43 for lookback windows of 3, 6, and 12, respectively, it’s clear that this parameter choice has a major impact on portfolio performance. Additionally, a 12-month look-back window produces positive alpha while 3 and 6 produce negative alpha on average. Recall that look-back window impacts the speed of our signal (how quickly we allow it to change month-to-month). These results seem to indicate that firm-specific information that can predict periods of persistent future returns exists as far back as 12 months, which is somewhat contrary to what I might have expected. Total return momentum is also traditionally built on a 12 month look-back window, so my prior was that residual momentum would be a faster moving signal because it uses the portion of returns which are idiosyncratic and therefore shouldn’t have long periods of persistence. And although it’s difficult to extract exactly the implication, it suggests that maybe short term fluctuations in a stock’s idiosyncratic return is more noise than it is information (at least in predicting periods of persistent future returns). Looking at the factor loadings, it’s clear that a longer look-back window does cause our residual momentum factor to load more heavily on total-return momentum risk - however, this could largely be due to the fact that I only have access to the 12-month total return momentum factor to run these regressions.

4.3 Portfolio Construction

Finally for our portfolio construction, one can immediately see a drastic difference in the equal vs. value weighted portfolio performances. Recall equal-weighted portfolios typically outperform due to loading on SMB/HML, because of the mechanics which cause them to take
contrarian views (counter-intuitive to momentum), or because of the relationship between idiosyncratic volatility and equal vs. value weighted portfolio returns. In the case of our residual return portfolios, the HML and SMB loadings of equal and value are both low and almost identical, so it’s not the first reason. It’s difficult to test the contrarian portion, but it’s still unclear why that would contribute at all (you would think it’d hurt). So we’re left with the possibility that high idiosyncratic volatility of the stocks in our portfolio is correlated with the equal outperforming the value weighted portfolio.

A simple way to test this is to see if the difference in returns to an equal vs. value weighted portfolio is correlated to the idiosyncratic risk of stocks within that portfolio. While ideally I’d do this across all portfolios, this is computationally very intensive, so we will look at a particular example. Namely, where the equal weighted portfolio most outperformed the corresponding value-weighted portfolio (all other parameters equal): 5-factor model with normalized residual returns using a look-back of 3 and a holding period of 1 month. Not surprisingly, we find support for this final explanation. Figure 3.1 shows the results of two regressions. The first (sans intercept) shows that on average, our equal weighted portfolio outperforms our value weighted portfolio more when idiosyncratic risk of our holdings increases. Our second regression (including intercept) shows that even assuming a base level of outperformance by equal over value, the difference is exacerbated as idiosyncratic volatility increases (though only significant at the 10% level). Although clearly this isn’t the entire explanation, given the significance of the intercept in the second regression, it’s clear that this is playing a role. Perhaps further research could explore this relationship more closely.

Moving on finally to holding period, it seems that longer holding periods perform better and enjoy greater diversification. A holding period of 1 does abysmally, which is not
out of the ordinary with momentum strategies as they rely mainly on multiple-period holding windows to capture the full autocorrelation of returns. As you move to 3, 6, and 12, you see continual improvement in portfolio performance, suggesting residual momentum can successfully predict periods of returns that are persistent up to 12 months into the future. This is fairly consistent with the behavior of total return momentum, though 6 months is fairly common-practice for many momentum investors. As far as a sanity check, our residual momentum appears to behave in a way that seems reasonable given the comparative statics and our intuition behind them. Moving forward, we will now examine a specific portfolio using the parameters outlined above which provided the best performance and diversification benefit: un-normalized 5-factor residuals rolled up over a 12-month look-back window for an equal-weighted 12-month holding period portfolio.

5 Portfolio Analysis

5.1 Portfolio Behavior

Before exploring the performance of the portfolio, let’s explore some of its behavior. First exploring industry composition, we can look at the first regression in Figure 8 which is a simple two-sample t-test on whether the loadings on each industry are significantly different between winner and loser portfolios. The results indicate that our winner portfolio seem to load more heavily on finance, and less heavily on non-durables and “other,” which is a category that encompasses quite a large portion of our stock universe given the few buckets. There isn’t a tremendous amount to read into which specific industries have different loadings, though the fact that some do seems to indicate that stocks in certain industries are more or less prone to exhibiting residual momentum.
Perhaps more interesting are the results in Figure 9, which summarizes the correlation between loadings between winners and losers. Namely, it compares for each industry: my current loading minus recent loading by my winners portfolio to my current minus recent loading in that industry for my losers portfolio. The negative coefficients indicate that if I’m loading more heavily than recently on a particular industry in my losers portfolio, I’m loading less heavily than recently on that industry in my winners portfolio. If industry had nothing to do with firm-specific momentum then there is no reason why our winner vs. loser portfolios should see any contemporaneous movement when categorized by industry, but here they do.

Perhaps part of what we’re capturing is industry momentum, which has been shown to exist just as significantly as on the individual stock level (Moskowitz, 1999). But this shouldn’t be possible given that our residuals are built by removing industry returns. Stocks from “winning industries” should have that portion of their returns excluded and therefore any industry-wide momentum should be controlled for. One interesting possibility is that certain members within an industry act as industry momentum leaders, predicting industry momentum before it becomes clear across all stocks in the industry. Our residual momentum strategy would recognize these stocks before an industry momentum strategy would, and trade accordingly – that said, our strategy may not capture all of the potential returns in that industry. If the aforementioned is really the case, then a strategy which traded industry momentum conditioned on residual momentum would be even more profitable. You build your residual momentum factor just as we did here, but instead of just picking up stocks with high residual momentum, you look for patterns in the industry loadings suggested by your residual momentum strategy and buy other stocks in the “nascent momentum” industries. Of course this is largely speculation and would require an entirely new extension of the given research, but it’s an interesting pursuit nonetheless.
Looking now at firm size, Figures 11 and 12 summarize the behavior of our portfolio using the same style of regressions as those for industry except now looking at average firm size of winners vs. losers. The t-test reveals that our winner portfolio on average is buying larger stocks than our loser portfolio, which mirrors what we find in total return momentum (Jegadeesh, 1993). The demeaned regression indicates that there isn’t a significant contemporaneous correlation between the firm size of winners and the firm size of losers. This is consistent with the fact that our SMB exposure is very low, and also means that the SMB factor does a relatively good job of extracting the portion of returns which express information of size related risk. Looking at the winner-losers plot in Figure 10 doesn’t reveal very much of interest in the early years, where the winner-loser behavior seems more or less random. In recent years there appear to be clear spikes up (winners being larger than losers) in 2000 and 2006, and then a major drop in 2010 – but plotted against this is the SMB cumulative return, which shows no unusual activity to the degree that would explain these spikes. If there was a strong reaction in the returns of the portfolio it would be worth exploring further, but as of now this is probably an area of distant future research.

5.2 Portfolio Performance

To analyze the performance of the portfolio we use fairly standard metrics in asset pricing. To begin, Figure 13 shows cumulative returns of this portfolio and some simple summary statistics. The Sharpe ratio of the portfolio is a simple metric for the risk adjusted returns of the portfolio, and is calculated as the average return of the portfolio over its volatility. An annual Sharpe of .67, while not phenomenal, is well above a threshold of considerable investment opportunities. That said, the statistic only gives an idea of the investment opportunity standalone, and reveals nothing about the systematic risk or possible
diversification benefit the portfolio provides. This is why alpha is typically a better measure of a portfolio’s success, as it represents the average outperformance of the portfolio relative to its expected return given its various risk exposures. In most cases, as is done in Figure 14, people use the 4-factor model as a benchmark, to which this portfolio has an annualized alpha of 4.8%. Recall that our empirical asset pricing models assume you should be compensated for the systematic risk that you take on, and any other returns should be idiosyncratic (captured in the residual). Alpha represents consistent returns generated by a portfolio that cannot be explained by its systematic risk exposures, and is therefore considered systematic outperformance.

Looking at figures 15 and 16 we see the factor exposure and alpha of the portfolio calculated on an expanding basis. This is a more realistic depiction of the portfolio’s factor exposure because A) it’s dynamic, and B) it doesn’t do any in-sample estimation. It’s the equivalent of rerunning the regression each day with all of the data you had up until that point. What you hope to see is convergence in your estimates as you gain more data, as it indicates a constant/predictable level of exposure to each factor. Our exposures to the first 3 factors (market beta, SMB, HML) all converge to ~0, which means our portfolio provides diversification benefit by producing returns without those returns being correlated with any of the factors. This is what we would hope to see, especially given the construction of our strategy.

Our momentum loading, while not alarmingly high at ~1.5-2, does exhibit a small but permanent jump around the 2000’s. This period marks the end of the dot com bubble, where both our residual momentum and total return momentum had sizeable downturns. If we think about what would cause residual momentum to mirror total return momentum, it’d be when
the factor model used to produce the residuals is unable to explain the returns of stocks in the economy. The portion of returns that can’t be explained by the factor model sinks entirely into the residual, and you end up with total returns and residual returns being very high for that stock, causing both momentum portfolios to pick them up. Although there isn’t any literature around the topic, it’s not farfetched to imagine that “dot-com” stocks at the time were behaving far from the expectation of the simple factor models. Of course in our factor model we also extract industry-wide returns, so if the entire tech industry was outperforming then our residuals wouldn’t suffer from the aforementioned inaccuracy. That said, it’s possible that in our data the industry specifications were such that many stocks other than the “dot-com” stocks fell into the “tech” category, which would dampen the overall industry returns and cause the problem mentioned previously. It would be an interesting vein of further research, but for my purposes it’s not of paramount importance.

And if it were just the tech bubble, then the correlation between residual and total return momentum would drop afterwards, however it appears to remain around .2 for the last ~15 years. The alpha of the portfolio (figure 16) has a permanent drop at the same time – this is partially a mechanical result of the fact that more of our portfolio’s returns are now being explained by its momentum exposure. It could also partially be the case that alpha in the stock market is generally disappearing over time, but this claim is essentially impossible to substantiate and there is no real consensus in the literature on this matter. That said, the drop in alpha is rather severe given the only 5% increase in exposure to momentum, and even if alpha was disappearing over time it probably wouldn’t do so in such a sudden manner. Another possible explanation is that our factor models used to construct our residuals became less accurate after 2000, causing our residuals to much more closely mirror our actual observed returns. However if we compare the average R² of our factor model, which is plotted against
alpha, to the performance in the last 15 years, it appears that this is not the case. A final explanation remains plausible, considering the fact that total return momentum has suffered over the same period: perhaps returns are losing their persistence, and therefore any momentum portfolio will suffer or perhaps have to adjust by using a shorter holding window. Whatever the reason, it’s clear that residual momentum has lost some ability to predict periods of persistent future returns in the last decade.

6 Conclusion

Residual momentum does behave in a manner that’s intuitive and provides diversification benefit with respect to our factors while still generating alpha. Its performance is quite sensitive to both the weighting scheme and look-back window, where equal-weighting far outperforms value-weighting and longer look-back windows outperform shorter ones. Although not as drastically as one might expect, the residual construction process reveals a potential tradeoff between performance and diversification benefit: the more factors used in residual construction, the lower are that portfolio’s factor loadings – however this comes with slightly weaker performance, at least as measured by Sharpe. That said, the alpha of the portfolio reveals that its diversification benefits far outweigh its relatively low standalone risk-adjusted returns.

Looking at a specific portfolio, we found that it consistently outperformed the benchmark 4-factor model, though suffered somewhat in the past 10-15 years along with total return momentum. This doesn’t appear to be due to lower levels of accuracy in our factor models, and in fact it appears that perhaps the opposite has happened in recent years. The fact that both total and residual return momentum suffer simultaneously suggests that
perhaps returns have lost some persistence in recent years, causing all momentum strategies to suffer. That said, residual return momentum can still be a valuable investment option for individuals with heavy loadings in the Fama French Factors, and perhaps as a substitute to traditional total return momentum.

One interesting result that surfaced was the apparent industry effect being captured by residual return momentum. Winner and loser portfolios have significantly different loadings in particular industries, and see negative contemporaneous correlation in their industry loadings as well. This suggests that residual returns might carry information which can be used to predict industry-wide momentum before it becomes clear for all stocks across that industry. Testing the behavior and performance of an industry momentum strategy that conditioned on residual returns would be a worth-while pursuit in further research.
7 Figures

7.1 Sensitivity Analysis

*Figure 1: Sensitivity Analysis Summary Table*

<table>
<thead>
<tr>
<th>Residual Extraction</th>
<th>Values</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
<td>3-Factor</td>
<td>Residuals are the portion of returns which are not explained by the factors -</td>
</tr>
<tr>
<td></td>
<td>4-Factor</td>
<td>momentum is therefore firm-specific</td>
</tr>
<tr>
<td></td>
<td>5-Factor</td>
<td></td>
</tr>
<tr>
<td><em>Factor Formation</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Look-back (J)</td>
<td>J = 3 months</td>
<td>Longer look-back means a slower/flatter momentum signal which takes more</td>
</tr>
<tr>
<td></td>
<td>J = 6 months</td>
<td>time/extreme moves to change directions</td>
</tr>
<tr>
<td></td>
<td>J = 12 months</td>
<td></td>
</tr>
<tr>
<td>Normalization</td>
<td>Untouched</td>
<td>If we believe residual returns are incomparable across time or across stocks,</td>
</tr>
<tr>
<td></td>
<td>Rolling 3-year Std</td>
<td>we should normalize</td>
</tr>
<tr>
<td><em>Portfolio Construction</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weighting</td>
<td>Equal-Weighted</td>
<td>Equal weight will put more weight into smaller firms. Can outperform because</td>
</tr>
<tr>
<td></td>
<td>Value-Weighted</td>
<td>of contrarian mechanic or idiosyncratic volatility relationship</td>
</tr>
<tr>
<td>Holding Period (H)</td>
<td>H=1</td>
<td>Longer holding period means our residual returns predict longer strings of</td>
</tr>
<tr>
<td></td>
<td>H=3</td>
<td>persistent future returns</td>
</tr>
<tr>
<td></td>
<td>H=6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>H=12</td>
<td></td>
</tr>
</tbody>
</table>
Sensitivity Analysis statistics are averaged across all portfolios that correspond to the parameter assignment in a given column. For example: in Figure 2, Sharpe-3 represents the average Sharpe across all portfolios that build residuals out of a 3-factor model. For each portfolio, Sharpe is calculated as the average return over average standard deviation of the portfolio, and represents a risk-adjusted return for the portfolio standalone. The remaining statistics are the coefficients from regressing each portfolio’s excess returns on the 4-factor model, with alpha being annualized. For all sensitivity analysis and portfolio performance figures, sharpe and alpha are calculated monthly and then annualized by multiplying by $\sqrt{12}$.

**Figure 2: Sensitivity Analysis of Residual Extraction**

<table>
<thead>
<tr>
<th>Factors</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.28</td>
<td>0.27</td>
<td>0.26</td>
</tr>
<tr>
<td>Alpha</td>
<td>-0.01</td>
<td>-0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Mkt Beta</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>HML</td>
<td>0.04</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>SMB</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>MOM</td>
<td>0.25</td>
<td>0.24</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Figure 3: Sensitivity Analysis of Factor Construction – Normalization**

<table>
<thead>
<tr>
<th>Norm</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>Alpha</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>Mkt Beta</td>
<td>-0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>HML</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>SMB</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>MOM</td>
<td>0.24</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Figure 3.1: Idiosyncratic Volatility Regression**

$$R_{Eq,t} - R_{Val,t} = [\alpha] + \beta \frac{1}{|S_t|} \sum_{s \in S_t} \sigma_{s,t-36} \quad S_t = \text{stocks} \in \text{portfolio at} \ t$$

<table>
<thead>
<tr>
<th>Beta</th>
<th>Coef</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.07</td>
<td>2.84</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef</td>
<td>0.01</td>
</tr>
<tr>
<td>T-Stat</td>
<td>7.84</td>
</tr>
</tbody>
</table>

We regress the difference between equal and value-weighted portfolio returns on the average volatility of stocks being held in the portfolio, calculated on a 36-month rolling window. The left regression excludes an intercept, while the right includes it.
### Figure 4: Sensitivity Analysis of Factor Construction – Look-back Window

<table>
<thead>
<tr>
<th>Look-back</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.10</td>
<td>0.28</td>
<td>0.43</td>
</tr>
<tr>
<td>Alpha</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Mkt Beta</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>HML</td>
<td>0.05</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>SMB</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>MOM</td>
<td>0.14</td>
<td>0.24</td>
<td>0.28</td>
</tr>
</tbody>
</table>

### Figure 5: Sensitivity Analysis of Portfolio Construction – Weighting

<table>
<thead>
<tr>
<th>Weight</th>
<th>Eq</th>
<th>Val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.50</td>
<td>0.04</td>
</tr>
<tr>
<td>Alpha</td>
<td>0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>Mkt Beta</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>HML</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>SMB</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>MOM</td>
<td>0.21</td>
<td>0.23</td>
</tr>
</tbody>
</table>

### Figure 6: Sensitivity Analysis of Portfolio Construction – Holding

<table>
<thead>
<tr>
<th>Holding</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.00</td>
<td>0.27</td>
<td>0.37</td>
<td>0.44</td>
</tr>
<tr>
<td>Alpha</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Mkt Beta</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>HML</td>
<td>0.10</td>
<td>0.06</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>SMB</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>MOM</td>
<td>0.24</td>
<td>0.24</td>
<td>0.22</td>
<td>0.18</td>
</tr>
</tbody>
</table>
7.2 Portfolio Behavior

**Figure 7: Winner vs. Loser Industry Composition**

These figures represent average loadings of the winner portfolio and loser portfolio in each industry averaged on a yearly basis. Other (purple), Finance (Brown), and Non-Durables (Green) are accented because winners and losers have significantly different loadings in these industries (summarized in figure 8).
Figure 8: Winner vs. Loser Industry Loading T-test

\[ i \in Industries : (W_{win}(i)_t - W_{lose}(i)_t) = \beta_i + \epsilon_t \]

\[ W_{pf}(i)_t = \text{Weight of portfolio } pf \text{ in industry } i \text{ at time } t \]

<table>
<thead>
<tr>
<th>T-Test</th>
<th>Manuf</th>
<th>Utils</th>
<th>Durbl</th>
<th>Other</th>
<th>NonDur</th>
<th>Retail</th>
<th>Energy</th>
<th>Finan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>0.004</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.016</td>
<td>-0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>0.013</td>
</tr>
<tr>
<td>T-Stat</td>
<td>1.52</td>
<td>0.24</td>
<td>0.55</td>
<td>-6.05</td>
<td>-2.42</td>
<td>0.82</td>
<td>0.07</td>
<td>6.51</td>
</tr>
</tbody>
</table>

Non-pooled regression done industry by industry. Betas can be interpreted as the average weight of our winner portfolio into each industry over the weight in that industry of our loser portfolio.

Figure 9: Winner vs. Loser Contemporaneous Correlation within Industries

\[ (W_{win}(i)_t - \overline{W_{win}(i)}_{3yr}) = \beta_i(W_{lose}(i)_t - \overline{W_{lose}(i)}_{3yr}) + \epsilon_t \]

\[ W_{pf}(i)_t = \text{Weight of portfolio } pf \text{ in industry } i \text{ at time } t \]

<table>
<thead>
<tr>
<th>Demeaned</th>
<th>Manuf</th>
<th>Utils</th>
<th>Durbl</th>
<th>Other</th>
<th>NonDur</th>
<th>Retail</th>
<th>Energy</th>
<th>Finan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>-0.60</td>
<td>0.02</td>
<td>-0.36</td>
<td>-0.24</td>
<td>-0.31</td>
<td>-0.54</td>
<td>-0.30</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Non-pooled regression done industry by industry. Betas can be interpreted as the weight change into a given industry for our winner portfolio given a 1 unit increase in the loading of our loser portfolio in that industry. W-bar is the average weight over the last 3 year in the given portfolio.
Figure 10: Average Firm Size in Winner vs. Loser Portfolio

This graph plots the average firm size in our winner portfolio, adjusted by weight, minus the average firm size in our loser portfolio, adjusted by weight. It is calculated independently for winners and losers (plotted in yellow and grey), and then the difference is plotted in blue. This is all plotted against the cumulative return of the SMB factor portfolio over the time period. The equation representation of our average firm size calculation for a given portfolio is as follows:

\[ S_{pf,t} = \sum_{i \in Stocks} w_{pf,t}(i) MktCap_{i,t} \]

Average firm size for portfolio \( pf \) at time \( t \) (\( S_{pf,t} \)) is calculated as the sum across all stocks of the weight that portfolio \( pf \) has in stock \( i \) at time \( t \) multiplied by the market cap of stock \( i \) at time \( t \).
Figure 11: Winner vs. Loser Average Firm Size T-Test

Beta is expressed as bps of total mkt cap. It can be interpreted as the difference in average firm size over average firm size of losers. Average firm size ($S_{\text{avg}}$) is calculated as in figure 9.

\[
(S_{\text{win},t} - S_{\text{lose},t}) = \beta + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
<th>T-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>0.02</td>
</tr>
<tr>
<td>T-Stat</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Figure 12: Winner vs. Loser Average Firm Size Contemporaneous Correlation

Beta can be interpreted as the change in average firm size of winners given a 1 unit change in average firm size of losers. Average firm size ($S_{\text{avg}}$) is calculated as in figure 9. S-bar represents the average firm size in the portfolio over the last 3 years.

\[
(S_{\text{win},t} - \bar{S}_{\text{win}3yr}) = \beta(S_{\text{lose},t} - \bar{S}_{\text{lose}3yr}) + \epsilon_t
\]

<table>
<thead>
<tr>
<th></th>
<th>Demeaned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>0.06</td>
</tr>
<tr>
<td>T-stat</td>
<td>1.57</td>
</tr>
</tbody>
</table>
7.3 Portfolio Performance

Figure 13: Cumulative Return and Portfolio Summary

![Cumulative Return Graph]

<table>
<thead>
<tr>
<th>Portfolio Summary</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharpe</td>
<td>0.67</td>
</tr>
<tr>
<td>Cum Ret</td>
<td>6.92</td>
</tr>
<tr>
<td>Avg Ret</td>
<td>0.08</td>
</tr>
<tr>
<td>Vol</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 14: Portfolio Alpha and Factor Exposures (Full-sample)

\[ R_{R_{\text{mom},t}} - R_{f,t} = \alpha + \beta_{Mkt}(R_{Mkt} - R_{f})_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{MOM}MOM_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>Mkt Beta</th>
<th>SMB</th>
<th>HML</th>
<th>MOM</th>
<th>Alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>0.04</td>
<td>0.03</td>
<td>0.06</td>
<td>0.19</td>
<td>0.048</td>
</tr>
<tr>
<td>T-Stat</td>
<td>2.07</td>
<td>0.97</td>
<td>1.83</td>
<td>13.12</td>
<td>3.78</td>
</tr>
</tbody>
</table>

This regression is run full-sample using monthly returns, and alpha is again annualized by multiplying by \( \sqrt{12} \).
Figure 15: Portfolio Factor Exposure (Expanding Window)

The same regression is run as in figure 7, only using an expanding window. Coefficients are plotted over time.

Figure 16: Portfolio Alpha vs. $R^2$ (Expanding Window)

Alpha is again annualized by multiplying by $\sqrt{12}$, and the $R^2$ is calculated as the average $R^2$ generated by our factor model to extract the residual returns to use in our factor. The first few years of data have their y-values censored for the sake of readability, and because those statistics are meaningless given the small sample used to calculate them.
8 References


