Why do Restaurants Offer Reservations?

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Abstract

This paper develops a game-theoretic model of restaurant policies, in which customers have noisy beliefs about their valuations at the time that they can choose to ask for reservations. I find conditions under which my model suggests that it is sensible for restaurants to offer reservations, and under which it is sensible to accept walk-in customers, as well as conditions under which customers will react in different ways to potential policies from the restaurant. I also show how this model is able to explain some of the real-world variation that we see in restaurants’ reservation and walk-in policies.

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1 Introduction

Restaurant reservations are an unusual economic practice. They provide a service to customers by guaranteeing they will have space at the restaurant, and they are potentially costly, since a restaurant may end up holding tables for “no-shows,” i.e. customers who have reservations but do not actually go to the restaurant. However, reservations are typically provided to customers for free. Recently there have been some restaurants that charge fees for reservations, such as Alinea, a restaurant in Chicago that requires that its customers pay the entire cost of their meal when making a reservation. However, restaurants like this are the exception, and this only happens in very high-end restaurants.

In general though, there is a great deal of variation in restaurants’ reservation policies. Fast-food restaurants, like McDonalds, do not offer reservations, while a very large number of “sit-down” restaurants do offer them, which might suggest that the benefits to a restaurant of offering reservations increase as the dining experience provided becomes more formal. However, there are also a number of high-end restaurants that take no reservations, requiring that anyone interested in eating at them waits in line.

This paper seeks to provide deeper insights into the source of this variation, as well as how customers respond differently in environments with and without the opportunity to make reservations. While it could be interesting to take a prescriptive route and design other mechanisms for this setting, the focus of this paper is to be descriptive and explain restaurant behavior given the typical structures that exist. In general, my model suggests that the optimal decision for a restaurant depends significantly on how likely it is that customers will be no-shows, and on how willing customers are to travel to the restaurant if they do not have reservations, since they may be turned away.
Section 2 provides a review of some of the existing literature, section 3 presents my model of restaurants, section 4 analyzes the model and finds equilibria, and section 5 goes through some examples of applying the model to real-world restaurants, using the results from section 4. I end with a discussion of the results and of potential future extensions.

2 Literature Review

There is an existing literature on the topic of advance sales. Png (1989) considers an environment with a seller that makes pricing and capacity decisions, and customers that can choose to purchase a reservation based on ex-ante beliefs of what their valuations will be at the time of consumption, and of what capacity will be available. Customers then can choose whether or not they wish to exercise their reservations and purchase the good once their valuation has been revealed, though if the seller has oversold reservations then only a subset of customers will actually get the good. It finds that reservations allow the seller to charge higher prices than would otherwise be feasible.

Su and Zhang (2009) models a related environment in which a seller chooses a price and quantity for a good, but buyers do not know the current quantity at any given time. The buyers must expend a search cost before they can know whether the good is still available and potentially buy it, but they are not able to make any sort of reservation. This is very similar to walk-in customers at a restaurant who do not know whether they will be able to get a seat. The paper finds that the seller benefits from using availability guarantees, e.g. offering customers a gift if they arrive and all of the good has been sold.

That said, restaurants have some particular features not captured by these models. For example, restaurants generally do not overbook; anyone who makes a reservation
and shows up at the proper time is quickly seated. Also, while availability guarantees are an interesting concept, they are not typically seen in the restaurant setting.

There is also a small existing game theory literature specifically on restaurant reservations. In fact, Alexandrov and Lariviere (2011) gives a model that shares many features with the one created in this paper. My model extends it in two significant ways, namely by focusing on an environment with two customers, rather than a mass of infinitesimal customers, and by allowing customers to have informed beliefs about how much they value going to a restaurant before they have to make a reservation. Modeling an environment with only two discrete customers is clearly a simplification, but it makes the problem far more tractable than allowing for an arbitrary number of customers. It also gives very different insight than a model with a continuum of customers, and I believe much of this insight would still apply to an environment with any number of discrete customers.

3 Model

I consider a single restaurant with a single table, and a market with 2 customers. I assume that the restaurant’s prices are fixed, and that the restaurant has the same profit from serving either customer. I also assume that the customers have types that differ based on their valuations for eating at the restaurant and their initial beliefs over what their valuations are, which I assume are accurate. Informally, consider a scenario in which a customer who wants to make a reservation must do so a week in advance. Such a customer is uncertain about whether she will want to eat at the restaurant a week after making the reservation; she may have a time conflict, or get sick, or simply decide that she is uninterested. However, she does still have initial expectations about how likely it is that she will want to eat at the restaurant.

Formally, each customer $i \in \{1, 2\}$ has type $t_i = (b_i, v_i) \in \{b_H, b_L\} \times \{-\epsilon, 1\}$,
where $v_i$ is the utility that the customer gets if she eats at the restaurant and where $b_i = Pr(v_i = 1|b_i)$. I assume $\epsilon > 0$, $b_H > b_L$, and that the customers’ types are i.i.d. from a distribution with $P_b = Pr(b_i = b_H), P_v = Pr(v_i = 1) = P_b b_H + (1 - P_b) b_L$.

The game has 3 periods.

- In period 1, the restaurant chooses whether or not it will accept reservations, and whether or not it will accept walk-ins if neither customer reserved the table. Formally, it publicly announces an action plan $(R, W) \in \{0, 1\}^2$, where $R = 1$ means it will accept a reservation, and $W = 1$ means it will accept a walk-in if there is space. It is costly for the restaurant to be open, so there will be a cost $c_O < 1$ that must be paid if $W = 1$ or if $R = 1$ and someone made a reservation.

- In period 2, each customer privately observes her own $b_i$. Each customer then has the option to request a reservation if $R = 1$. This is a costly action, with a cost of $c_r < 1$, representing the effort involved in making a reservation. Note that this cost is not paid to the restaurant. Any customer who requested a reservation learns immediately whether or not they got it, and if both customers request a reservation then it is allocated randomly. A customer who asks for a reservation but does not get it knows that this means the other customer did get it. Customers who did not request a reservation do not observe whether others did. Formally, each customer $i$ responds to $(R, W)$ and $b_i$ by choosing an action $a^i \in \{0, 1\}$ where $a^i = 1$ means that $i$ asks for a reservation. I allow customers to choose behavioral strategies, so I will use $\alpha^i \in [0, 1]$, to represent the probability with which $i$ asks for a reservation. I will often refer to the probability when $b_i = b_H$ as $\alpha^i_H$, and similarly for $\alpha^i_L$. I will also denote by $\alpha^i_T = \alpha^i_H P_b + \alpha^i_L (1 - P_b)$ the probability that $i$ asks for a reservation if her belief is unknown — e.g. from the restaurant’s or the other customer’s perspective. Technically these are
functions and should be denoted $\alpha_H(R, W)$, but I will generally drop the $R$ and $W$ when their values are clear from context.

- In period 3, each customer privately observes her own $v_i$. Customers with reservations decide whether or not to go to the restaurant, and if $W = 1$ then customers who did not request reservations decide whether or not to try to walk in. If both customers walk in, then again the table is allocated randomly. If a customer walks in and is turned away, either because the other customer had reserved the table — even if they were a no-show — or because the other person was randomly given it, she faces a cost $c_N$. This cost can be thought of as including things like disappointment, or the cost of having to adjust plans. Also, it can include things like travel costs; while these will be expended even if the customer does get the table, we can think about $v_i$ as already taking them into account.

Formally each customer chooses an action $g_i \in \{0, 1\}$, where $g_i = 1$ means that $i$ goes to the restaurant. Again, I allow for behavioral strategies so I will use $\gamma_i$ to represent the probability with which $i$ goes to the restaurant. Note that low-valuation customers will never go to the restaurant since they get a disutility from eating. Also, customers who asked for a reservation but did not receive one will never go since they are guaranteed to be turned away. Finally, high-valuation customers with reservations will certainly go since they are guaranteed a table. Thus $\gamma_i$ is only of interest in the case of high-valuation customers who did not request reservations. Because of this, I am treating $g_i$ and $\gamma_i$ as values to save on notation, even though they should technically be functions. I also restrict $\gamma_i$ to not depend on $b_i$ since there is no clear justification for a customer’s strategy being dependent on prior, outdated beliefs.
At the end of the game, the payoff to the restaurant is

\[
u_R = \begin{cases} 
1 - c_O & \text{if a customer is served} \\
-c_O & \text{if the restaurant stays open and no customers are served} \\
0 & \text{if the restaurant is not open (i.e. if } W = 1 \text{ and no customers made reservations)}
\end{cases}
\]

Note that this allows for variation in risk aversion for the restaurant. There are only three possible outcomes and the restaurant’s preference ordering between them is stable in \( c_O < 1 \), and \( \frac{1-c_O}{c_O} \) can take any value in \( \mathbb{R}_{>0} \). Increasing \( c_O \) and increasing risk aversion would have the same effect on this ratio, and thus we can capture changes in either by adjusting \( c_O \).

The payoff for customer \( i \) is

\[
u_i = v_i \mathbb{1}_{EAT} - c_r \mathbb{1}_{RESERVED} - c_n \mathbb{1}_{TURNED AWAY},
\]

where the indicator functions, in order, denote whether \( i \) ate at the restaurant, asked to reserve a table, and walked in but was turned away. Implicit in this function is the assumption that the customers are risk neutral.

\section{Analysis}

In this section I characterize the equilibria of the model. I constrain the analysis to perfect Bayesian equilibria, specifically the stronger notion of Fudenberg and Tirole (1991). In particular, beliefs satisfy the “no signaling what you don’t know” condition. As a result, since the restaurant does not know the types of the customers, the customers’ initial equilibrium beliefs about the other players must be the same regardless of what the restaurant chooses to do, and thus they must coincide with the distributions determined by nature. Since the customers never observe anything about each other until after they have made their final decisions, this means we can apply Bayes’ rule at all information sets.

I also constrain the analysis to equilibria that are symmetric for the customers,
since these require no coordination between the customers. I will often drop the $i$ subscript or superscript, and add a $*$ superscript — e.g. $\gamma^*$ — when referring to a symmetric best response for the customers.

Due to the dynamic nature of this game, we can alternatively think about it as the restaurant choosing between four different games for the customers, $(R, W) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. Because of this, I take the general approach of characterizing the best response strategies for the customers for any selection of $(R, W)$, and then looking at which yields the highest expected payoff for the restaurant.

### 4.1 Restaurant Payoffs

I first consider what the payoffs to the restaurant will be, given a choice of $(R, W)$ and a symmetric response from the customers, $\alpha^*_H, \alpha^*_L, \gamma^*$.

**Theorem 1.** The expected payoffs to the restaurant, depending on $(R, W)$, are

- $\mathbb{E}(u_r(0, 0)) = 0$
- $\mathbb{E}(u_r(0, 1)) = 1 - c_O - (1 - \gamma^*(0, 1)P)\)\]
- $\mathbb{E}(u_r(1, 0)) = (2 - \alpha^*_T(1, 0))(P\alpha^*_H(1, 0)(b_H - c_O) + (1 - P)\alpha^*_L(1, 0)(b_L - c_O)$
- $\mathbb{E}(u_r(1, 1)) = \mathbb{E}(u_r(1, 0)) + (1 - c_O)(1 - \alpha^*_T(1, 1))^2$
- $\quad - (P(1 - \alpha^*_H(1, 1))(1 - b_H\gamma^*(1, 1))$
- $\quad + (1 - P_b)(1 - \alpha^*_L(1, 1))(1 - b_L\gamma^*(1, 1))^2$

**Proof.** This is a result of algebraic manipulation without any significant insights, so I omit the details of the proof. \qed
4.2 Customer Responses

I now focus on the symmetric best response strategies of the customers to any choice by the restaurant.

I begin with some trivial observations.

**Observation.** If $R = 0$ then $\alpha^*_H = \alpha^*_L = 0$. If $W = 0$ then $\gamma^* = 0$.

**Proof.** If $R = 0$ then the restaurant accepts no reservations, so the customers will never request them since they are costly. Similarly, if $W = 0$ then customers will never walk in since they will certainly be turned away. \hfill \square

**Observation.** If $R = W = 0$ then $\alpha^*_H = \alpha^*_L = \gamma^* = 0$. Also, $u_r = u_1 = u_2 = 0$.

**Proof.** This is clear from the previous observation and from the model setup. \hfill \square

Next, I consider how customers respond to other choices by the restaurant, starting with only taking walk-ins.

**Theorem 2.** If $(R, W) = (0, 1)$ then $\gamma^* = \min\left\{1, \frac{2}{P_v(1 + cN)}\right\}$.

**Proof.** No customers are allowed to make reservations, but they can walk in. Suppose that customer $j$ is going with probability $\gamma_j$. Customer $i$ will go to the restaurant when $v_i = 1$ and

$$0 \leq 1 \cdot Pr(i \text{ gets table}) - c_N \cdot Pr(i \text{ does not get table})$$

$$= (1 - \gamma_j P_v/2) - c_N(\gamma_j P_v/2)$$

$$= 1 - \gamma_j(c_N P_v/2)$$

i.e. when $\gamma_j \leq \frac{2}{P_v(1 + c N)}$. The same holds for when customer $j$ will go, and since $\gamma$ represents a probability and thus can be at most 1, we have that the only symmetric best response is $\gamma^* = \min\left\{1, \frac{2}{P_v(1 + cN)}\right\}$. \hfill \square
Next I consider what happens when the restaurant takes reservations. I begin with two lemmas.

**Lemma 1.** Suppose $\gamma^* < 1$ is a symmetric best response to $(R, W)$. Then the expected continuation payoff starting in period 3 for any high-valuation customer who did not request a reservation is 0.

*Proof.* Suppose $\mathbb{E}(u_t(g_i = 1)) < 0$. Then customer $i$ will choose $\gamma_i = 0$, yielding a continuation payoff of 0. Now suppose $\mathbb{E}(u_t(g_i = 1)) > 0$. Then $i$’s best response is $\gamma_i = 1$, contradicting the assumption that $\gamma^* < 1$. \qed

**Lemma 2.** If there is an equilibrium with $(R, W) = (1, 1)$ and $\gamma^* = 1$, then it is not a unique equilibrium.

*Proof.* Suppose there was such an equilibrium. It is clear that the probability of a given customer getting a table by walking in when $(R, W) = 1$ is lower than the probability when $(R, W) = (0, 1)$. Thus, we must have $\gamma^* = 1$ when $(R, W) = (0, 1)$. Then $\mathbb{E}(u_r(0, 1)) \geq \mathbb{E}(u_r(1, 1))$ since when $R = 0$ there is a no chance of reserving a table to a no-show. Therefore, we also have an equilibrium with $(R, W) = (0, 1)$ and $\gamma^* = 1$, so the above equilibrium is not unique. \qed

As an aside, in most cases the strategy profile described in Lemma 2 is not an equilibrium at all. If it were and there was a nonzero chance of reserving a table for a no-show, which would be true for many values of the parameters, then the strategy profile with $(R, W) = (0, 1)$ has a strictly higher expected payoff for the restaurant and will thus be preferred.

Because this is never a unique equilibrium and is often not an equilibrium at all, I will assume that when it is an equilibrium the restaurant always chooses $(R, W) = (0, 1)$ instead.

Next I prove a statement formalizing the intuitive notion that if customers with high beliefs do not strictly want to request reservations, then customers with low
beliefs will never want to request reservations. By the contrapositive, we get that if customers with low beliefs weakly want to request reservations, then customers with high beliefs strictly want to do so.

**Proposition.** Suppose $R = 1$ and $\gamma^* \neq 1$. If $\alpha^*_H \neq 1$ then $\alpha^*_L = 0$.

**Proof.** Since $\gamma^* \neq 1$ we see that the period-3 continuation payoff for both customers must be 0. $\alpha^*_H \neq 1$ implies that $\mathbb{E}(u_i(\alpha^*_H = 1)) = \mathbb{E}(u_i(\alpha^*_H = 0)) = 0$ We then have $\mathbb{E}(u_i(\alpha^*_i = 1)) = b_L(1 - \alpha^*_T/2) < b_H(1 - \alpha^*_T/2) = 0 = \mathbb{E}(u_i(\alpha^*_L = 0))$. Thus $\alpha^*_L = 0$. \qed

**Remark.** Note that one consequence of this is that in any equilibrium with $\alpha^*_T \geq P_b$ it must be the case that $\alpha^*_H = 1$. Similarly, if $\alpha^*_T \leq P_b$ it must be the case that $\alpha^*_L = 0$.

As a result, in any symmetric equilibrium with $R = 1$ and $\gamma^* \neq 1$ we can specify the values for $\alpha^*_H, \alpha^*_L$. The following theorem fully characterizes the best responses when $(R, W) = (1, 0)$ and characterizes the reservation portion of the best responses when $(R, W) = (1, 1)$ and $\gamma^* \neq 1$.

**Theorem 3.** Suppose $R = 1$, $\gamma^* \neq 1$. Then we have

$$(\alpha^*_H, \alpha^*_L) = \begin{cases} (1, 1) & 2 - \frac{2c_r}{b_L} \geq 1 \\ \left(1, \frac{2 - \frac{2c_r}{b_L} - P_b}{1 - P_b}\right) & 2 - \frac{2c_r}{b_H} \geq P_b \text{ and } 1 > 2 - \frac{2c_r}{b_L} \geq P_b \\ (1, 0) & 2 - \frac{2c_r}{b_H} \geq P_b \text{ and } 2 - \frac{2c_r}{b_L} < P_b \\ \left(2 - \frac{2c_r}{b_H}, 0\right) & 0 < 2 - \frac{2c_r}{b_H} < P_b \\ (0, 0) & 2 - \frac{2c_r}{b_H} < 0 \end{cases}$$

**Proof.** Suppose that customer $j$ is requesting a reservation with probability $\alpha^*_j$. Note that this is the probability customer $i$ would assign to the event, since $i$ does not
know $b_j$. Recall that by Lemma 1, the expected continuation payoff if player $i$ does not request a reservation is 0. Then we can see that $i$ will request a reservation when

$$0 \leq \mathbb{E}(u_i(a_i = 1|\alpha^j, b_i)) = 1 \cdot b_i(1 - \alpha^j/2) - c_r \implies \alpha^j \leq 2 - \frac{2c_r}{b_i}$$

If $2 - \frac{2c_r}{b_L} \geq 1$ then $i$ will always want to request a reservation, no matter what $j$ is doing. This is clear when $b_i = b_L$ and by the Proposition, $i$ must want to reserve when $b_i = b_H$. Symmetrically, $j$ will always want to reserve, regardless of what $i$ does. Thus we have case 1.

Now suppose $2 - \frac{2c_r}{b_H} \geq P_b$. Then $\alpha^*_T \geq P_b$ so we must have $\alpha^*_H = 1$. If $2 - \frac{2c_r}{b_L} < P_b$ then $i$ will not want to reserve when she has low beliefs, so $\alpha^*_L = 0$, and thus we have case 3. If $2 - \frac{2c_r}{b_L} \geq P_b$ then $i$ with low beliefs will be exactly indifferent when

$$\alpha^j_T = 2 - \frac{2c_r}{b_L} \implies \alpha^j_L = \frac{2 - \frac{2c_r}{b_L} - P_b}{1 - P_b}$$

since $\alpha^j_H$ must be 1. Thus we have case 2.

Finally suppose $2 - \frac{2c_r}{b_H} < P_b$. Then $\alpha^*_T < P_b$ so we must have $\alpha^*_L = 0$. If $2 - \frac{2c_r}{b_H} < 0$ then $i$ never wants to reserve even when she has high beliefs, so $\alpha^*_H = 0$ and thus we have case 5. Otherwise, $i$ with high beliefs is indifferent about reserving when

$$\alpha^j_T = 2 - \frac{2c_r}{b_H} \implies \alpha^j_H = \frac{2 - \frac{2c_r}{b_H}}{P_b}$$

since $\alpha^j_L$ must be 0. Thus we have case 4.

Finally, I characterize the $\gamma^*$ portion of the best response for the customers when $(R, W) = (1, 1)$. 

\[ \square \]
**Theorem 4.** Suppose \((R, W) = (1, 1)\) and \(\gamma^* \neq 1\). Then

\[
\gamma^* = \begin{cases} 
0 & \text{if } 0 > 1 - \alpha^*_T - c_N \alpha^*_T \\
\frac{2}{1+c_N} - 2\alpha^*_T & \text{otherwise}
\end{cases}
\]

where \(\alpha^*_H, \alpha^*_L, \alpha^*_T\) are the values determined by Theorem 3.

**Proof.** \(\gamma^* \neq 1\) means that the customers do not strictly prefer to go to the restaurant when they have high valuation but did not ask for reservations. Thus, they either strictly prefer not to go, or they are indifferent.

First suppose they strictly prefer not to go. Then clearly \(\gamma^* = 0\). This happens when

\[
0 > \mathbb{E}(u_i(g_i = 1)) = 1 \cdot Pr(i \text{ gets a table}) - c_N \cdot (1 - Pr(i \text{ gets a table}))
\]

where, since \(\gamma^* = 0\)

\[
Pr(i \text{ gets a table}) = 1 - \alpha^*_T - \frac{\gamma^*(P_b(1 - \alpha^*_H)b_H + (1 - P_b)(1 - \alpha^*_L)b_L)}{2}
\]

\[
= 1 - \alpha^*_T
\]

So this happens when \(0 > 1 - \alpha^*_T - c_N \alpha^*_T\).

Now suppose that when the customers use the best response strategy, they are indifferent about going to the restaurant. This means \(0 = \mathbb{E}(u_i(g_i = 1))\), and solving for \(\gamma^*\) we get the result in the statement of the theorem.

Combining Theorem 1 with the results of Theorems 2, 3, and 4 yields an optimal choice \((R^*, W^*)\) for the restaurant, and thus an equilibrium for the game. Below is a summary of the results.
Restaurant Payoffs

- $\mathbb{E}(u_r(0, 0)) = 0$
- $\mathbb{E}(u_r(0, 1)) = 1 - c_O - (1 - \gamma^*(0, 1)P_v)^2$
- $\mathbb{E}(u_r(1, 0)) = (2 - \alpha^*_T(1, 0))(P_b\alpha^*_H(1, 0)(b_H - c_O) + (1 - P_b)\alpha^*_L(1, 0)(b_L - c_O))$
- $\mathbb{E}(u_r(1, 1)) = \mathbb{E}(u_r(1, 0)) + (1 - c_O)(1 - \alpha^*_T(1, 1))^2 - (P_b(1 - \alpha^*_H(1, 1))(1 - b_H\gamma^*(1, 1)) + (1 - P_b)(1 - \alpha^*_L(1, 1))(1 - b_L\gamma^*(1, 1)))^2$

Customer Responses to $(R, W) = (0, 0)$

- $\alpha^*_T = 0$
- $\gamma^* = 0$

Customer Responses to $(R, W) = (0, 1)$

- $\alpha^*_T = 0$
- $\gamma^* = \min\left\{1, \frac{2}{P_v(1 + c_N)}\right\}$

Customer Responses to $(R, W) = (1, 0)$

- $(\alpha^*_H, \alpha^*_L) = \begin{cases} (1, 1) & 2 - \frac{2c_r}{b_L} \geq 1 \\ \left(1, \frac{2 - 2c_r}{b_L} - P_b\right) & 2 - \frac{2c_r}{b_H} \geq P_b \text{ and } 1 > 2 - \frac{2c_r}{b_L} \geq P_b \\ \left(2 - \frac{2c_r}{b_H}, 0\right) & 0 < 2 - \frac{2c_r}{b_H} < P_b \\ (0, 0) & 2 - \frac{2c_r}{b_L} < 0 \end{cases}$

- $\gamma^* = 0$
Customer Responses to \((R, W) = (1, 1)\)

- \((\alpha_H^*, \alpha_L^*) = \begin{cases} 
(1, 1) & 2 - \frac{2c_r}{b_L} \geq 1 \\
\left(1, \frac{2 - \frac{2c_r}{b_L} - P_b}{1 - P_b}\right) & 2 - \frac{2c_r}{b_H} \geq P_b \text{ and } 1 > 2 - \frac{2c_r}{b_L} \geq P_b \\
(1, 0) & 2 - \frac{2c_r}{b_H} \geq P_b \text{ and } 2 - \frac{2c_r}{b_L} < P_b \\
\left(\frac{2 - \frac{2c_r}{b_H}}{P_b}, 0\right) & 0 < 2 - \frac{2c_r}{b_H} < P_b \\
(0, 0) & 2 - \frac{2c_r}{b_H} < 0
\end{cases}\)

- \(\gamma^* = \begin{cases} 
0 & \text{if } 0 > 1 - \alpha_T^* - c_N \alpha_T^* \\
\frac{2}{1 + c_N} - 2\alpha_T^* & \frac{P_b(1 - \alpha_H^*)b_H + (1 - P_b)(1 - \alpha_L^*)b_L}{P_b(1 - \alpha_H^*)b_H + (1 - P_b)(1 - \alpha_L^*)b_L} & \text{otherwise}
\end{cases}\)

5 Example

The model is fairly complicated and there are large, discrete jumps between equilibria. Because of this, rather than doing general comparative statics I will present potential parameter values for real-world types of restaurants, and observe how behavior differs as a result of the different parameters. I will consider three types of restaurants: fast-food restaurants, fancy restaurants, and trendy restaurants. Below are descriptions of potential “average” restaurants and their customers in order to justify my choices of representative parameters. I then suggest specific values and finally compare the resulting equilibria.

5.1 Descriptions

Fast-food The value of eating fast food is probably low, so \(c_r\) and \(c_N\) should be high, since they are defined relative to the value, which is normalized to 1. That said, there is also most likely a very low cost to being turned away and eating somewhere
else, so this probably on net results in a low \(c_N\). The cost of ingredients and wages are also low, so \(c_O\) should be low.

Since I am assuming the value of eating at the restaurant is relatively low, this also suggests that \(b_H\) and \(b_L\) should be low since it is more likely that better options arise between the time customers would have to make reservations and when they would choose whether or not to go to the restaurant. However, fast food is typically cheap, so \(b_H\) and \(b_L\) should not be too low since fast food could be a back-up for customers who don’t find anything better. I will thus assume middling values for both, and I will also assume a middling value for \(P_b\).

**Fancy** The value of eating at a fancy restaurant may be high, due to higher quality food and atmosphere. As a result, \(c_r\) and \(c_N\) should be somewhat low. However, it is likely more costly to not get a table; for example, it may be very disappointing to plan to celebrate an anniversary at a fancy restaurant, only to be turned away after arriving. In addition, some potential customers might face high travel costs to reach the restaurant. Thus it seems that on net \(c_N\) should be high. \(c_O\) should also be high relative to the fast-food case, since fancy restaurants typically use more expensive ingredients.

Here, I would expect \(b_H\) to be very high since many potential customers might be interested in attending the restaurant for special events that have a prespecified time, e.g. meeting a business client, or celebrating an anniversary. For other customers, I would expect \(b_L\) to be somewhat low. I would also expect \(P_b\) to be somewhat high since many people only go to fancy restaurants for special occasions.

**Trendy** Unlike fast-food and fancy restaurants, “trendy” is a less typical restaurant description, so I will provide some real-world examples. Little Serow, a fancy Thai restaurant in Washington, D.C.; and Franklin Barbecue, a barbecue restaurant in Austin, TX, are two restaurants that are known for being extremely popular. At
both, it is common for customers to have to wait for multiple hours in order to get food because the lines are so long.

The value at this sort of restaurant should be very high since this is what makes the restaurant trendy and popular. Thus, $c_r$ and $c_N$ should be very low. Because the restaurant is trendy, people may be willing to face higher travel costs, but I will assume the value is still relatively low due to the high perceived value. I will assume that $c_O$ is somewhere in between the values for the fast-food and fancy restaurants.

I would expect both $b_H$ and $b_L$ to be high, though perhaps not as high as $b_H$ was in the fancy restaurant case.

### 5.2 Parameter Choices

**Fast-food** $(c_r = 3/5, c_N = 1, c_O = 1/5, b_H = 3/5, b_L = 2/5, P_b = 1/2)$

**Fancy** $(c_r = 1/4, c_N = 8, c_O = 1/2, b_H = 9/10, b_L = 2/5, P_b = 4/5)$

**Trendy** $(c_r = 1/10, c_N = 1, c_O = 1/3, b_H = 4/5, b_L = 3/5, P_b = 1/2)$

### 5.3 Resulting Equilibria

We can plug the above parameters into the results from Section 4.

**Fast-food** With these parameters, no one is ever willing to reserve, even if they have high beliefs. The equilibrium set of strategies is for the restaurant to only accept walk-ins, and for all customers with high value to attempt to walk in. This seems to fit with what we do observe in real fast-food restaurants.

**Fancy** With these parameters, it is actually worse for the restaurant to only take walk-ins than for it to not open at all. The equilibrium set of strategies is for the
restaurant to only offer reservations, and for only the high belief customers to request reservations. This also seems to fit with real-world observations, since fancy restaurants do typically offer reservations and can be very difficult to get a table at otherwise.

**Trendy** With these parameters, even low belief customers are always willing to request reservations if they have the option to do so. The equilibrium set of strategies is for the restaurant to only allow walk-ins, and for all customers with high value to attempt to walk in. This seems to fit with the existence of very popular and sometimes high-end restaurants that do not take reservations and are known for constantly having long lines.

### 5.4 Summary of Examples

From these examples, we can see that the model is able to capture typical real-world behavior for a variety of types of restaurants, as well as explain the reasons for the differences in their behavior. While this model is certainly a very simplified version of reality, it is capable of providing insights.

### 6 Discussion and Conclusion

This model provides potential insights into the question of why restaurants offer reservations, and why we see significant variation in reservation policies among restaurants. Offering reservations can be very beneficial for a restaurant in that it can encourage customers to come to the restaurant by guaranteeing them tables. However, reservations can also end up being costly to the restaurant by limiting its ability to allocate a table when there are “no-shows.”

This model does lack parameters that can adjust the ratio between the number of customers and the number of seats available. A model that incorporates such parame-
ters could provide additional, valuable insights into the variation among restaurants. Intuitively, it seems almost certain that this ratio would significantly impact customers’ behavior, and thus the optimal decisions for the restaurant, so it would be interesting to see an extension to this model that incorporates this. That said, the model is already very complex, and it is unclear how tractable it would be with an arbitrary number of customers and tables. A model with a mass of infinitesimal customers would be tractable, but that has its own limitations, particularly due to the law of large numbers eliminating a great deal of the uncertainty that is important in a model of reservations.

It would also be interesting to extend the model to handle multiple seating periods. Again this would make the model significantly more complex, but it would capture other effects of reservation policies. This paper focused on the insurance aspect of using reservations in order to attract more customers, but reservations may also act as a means of smoothing demand over the course of a night, rather than having separate periods of high and low demand.

General ideas from the model presented here can also be applied to some other environments. Many markets have capacity constraints, due to physical building constraints, production speed limitations, or other factors. These constraints certainly influence customers’ behavior, as they do in the model from this paper.
References


