Unilateral Environmental Policy in a Two-Country World with Directed Technological Progress *

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1 Introduction

Despite consensus in the academic community that climate change is real, man-made and costly, attempts at global policy coordination have failed. In 1997, the 83 signatories\(^2\) of the Kyoto Protocol agreed to reduce greenhouse gas emissions during the period from 2008 to 2012 by 7 percent on average relative to their 1990 levels. However, the United States and Canada both withdrew very early and many middle income countries believed that the Protocol would be an obstacle to their economic goals. By 2012, it covered just over one-fifth of the world’s total emissions. An extension to the agreement has been proposed but not ratified and is not in effect. Moreover, the Protocol was

\(^1\)See, for example, Chapters 6 and 24 in Nordhaus, William D. The Climate Casino: Risk, Uncertainty, and Economics for a Warming World. Yale UP, 2013. Print.

“ineffective because [it] provided no incentives to encourage participation. Countries have strong incentives to free ride on the efforts of others because emissions reductions are local and costly, while the benefits are diffuse and distant over space and time.”

In 2005, the European Union introduced the Emissions Trading Scheme (ETS) in 31 member states. The ETS, which is still in effect today, is a cap and trade program. By placing a cap on total emissions, requiring that commercial polluters hold permits consistent with the amount they emit, and creating a market for tradable permits, the program discourages greenhouse gas emissions by raising the price of polluting. However, when the ETS raises the price of carbon emissions in the EU relative the prices elsewhere in the world, EU-based business will weigh the costs of paying to pollute against the cost of relocating some or all of their operations outside the ETS’s area of effect. The more expensive it is to pollute in the EU relative to other areas, the more businesses will find it advantageous to relocate, other things being equal. In fact, there is evidence in the United Kingdom that greenhouse gas emissions as a result of domestic consumption have been flat since the advent of the ETS, even as emissions from domestic production have fallen.

To prevent this “leakage,” the obvious solution would be to extend the geographic boundaries of the program. If the ETS or another program that raised the price of emissions had a wide area of effect or covered most of the world’s economically developed regions, then it would be more expensive for business to relocate their most emissions-intensive operations outside the program’s jurisdiction. Unfortunately, no international cooperative effort to raise the global price of carbon has been forthcoming, and it is costly to wait.

3 Nordhaus, Chapter 21
(2012) (hereafter, AABH) and Nordhaus (2013) argue that the costs of climate change are likely to eclipse the costs of a well-designed policy response and that the longer we wait to introduce effective policies, the more costly that intervention will be. Given this urgency, we should re-consider the possibility of unilateral action.

Is it possible for a single nation or a regional body to significantly reduce global greenhouse gas emissions? In this paper, I will show that in a 2-country world without trade, an environmental disaster is inevitable, but that when trade is permitted, one country acting alone can prevent a disaster. This paper extends the one country, two-sector endogenous growth model with environmental constraints developed in AABH to a two-country “North-South” model. Throughout the essay, I use the name “North” for the country that wants to increase welfare by preventing harm to the environment, while the “South” lets the market operate freely. In both countries, household utility depends on consumption and environmental quality, and perfectly competitive firms produce the same consumption good using CES production functions with clean and dirty inputs. The damage to the environment each period is proportional to the quantity of dirty inputs produced. Inputs are made by perfectly competitive firms according to Cobb-Douglas production functions that combine labor with a continuum of sector-specific machines. Machines are supplied by monopolistically competitive firms that use the final good as their only input. In each period, scientists choose to conduct research (at no cost) in either the clean sector or the dirty sector by making an improvement to one of the machines in that sector. Successful scientists receive one-period patents and become the monopolists.

The paper proceeds as follows. Section 2 formally introduces the model. In Section 3, I characterize the equilibrium in the South under autarky and discuss the environmental implications. Section 4 introduces trade in inputs, derives a sufficient condition for preventing an environmental disaster, and describes a policy. Section 5 presents the conclusions and offers suggestions for future research.
2 Framework

The framework that follows is very similar to the one developed in AABH. This version excludes assumptions from the original when they are unnecessary, and makes new assumptions and notation changes to allow for two countries.

2.1 Consumer Preferences

Consider two discrete-time economies inhabited by households comprising workers, entrepreneurs, and scientists. Assume that in economy $k \in \{N, S\}$, there is an infinitely-lived, representative household whose utility in period $t \in (0, \infty)$ is an increasing, twice-differentiable function of consumption of the unique final good in period $t$, $C_{k,t}$, and environmental quality in period $t$, $S_t \in [0, \bar{S}]$. $\bar{S}$ is the quality of the environment in the absence of any human pollution. Further, assume that $u(C, S)$ is jointly concave in $(C, S)$ and satisfies

$$\lim_{C \to 0} \frac{\partial u(C, S)}{\partial C} = \infty, \lim_{S \to 0} \frac{\partial u(C, S)}{\partial S} = \infty$$  

(1)

An environment whose quality level has reached $S = 0$ is uninhabitable and should have a severe negative effect on utility, which is represented by assuming

$$\lim_{S \to 0} u(C, S) = -\infty$$  

(2)

The welfare of the representative household in country $k$ is the sum of discounted utility over its (infinite) lifetime:

$$\sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} u(C_{k,t}, S_t)$$  

(3)

where $\rho > 0$ is the household’s discount rate.
2.2 Production

The final good is produced by many firms using two intermediate goods according to a production function that exhibits constant elasticity of substitution between inputs:

\[ Y_{k,t} = \left[ Y_{k,c,t}^{\frac{\epsilon_k-1}{\epsilon_k}} + Y_{k,d,t}^{\frac{\epsilon_k-1}{\epsilon_k}} \right]^{\frac{\epsilon_k}{\epsilon_k-1}} \]  

where \( Y_{k,t} \) is the quantity of the final good produced in country \( k \) in period \( t \); \( Y_{k,c,t} \) and \( Y_{k,d,t} \) are respectively the quantities of the “clean” and “dirty” capital goods used to produce the final good in country \( k \) in period \( t \); and \( \epsilon_k \in (0, \infty) \) is the elasticity of substitution between the two intermediate goods in country \( k \). The usual distributional parameter is suppressed for notational simplicity. Producers sell the final good in a perfectly competitive market and buy the capital goods in perfectly competitive markets.

The capital goods are gross substitutes if \( \epsilon_k > 1 \) and gross complements if \( \epsilon_k < 1 \). As in AABH, I will ignore the Cobb-Douglas case where \( \epsilon_k = 1 \). Based on existing empirical work, it seems more reasonable to assume that the inputs are gross substitutes,\(^7\) so the analysis will focus on the part of the parameter space in which \( \epsilon_k > 1 \).

The capital goods are both produced using labor and a continuum of sector-specific machines. In both sectors, some machines are better than others; a better machine produces more of the capital good per period, for any given amount of labor used. For simplicity, assume that the capital goods fully depreciate in one period so that, in every period, the number of capital goods produced is equal to the capital stock. The quantity of the capital good in sector \( j \) in country \( k \) in period \( t \), \( Y_{k,j,t} \), is given by:

\[ Y_{k,j,t} = L_{k,j,t}^{1-\alpha_k} \int_0^1 A_{k,j,i,t}^{1-\alpha_k} x_{k,j,i,t}^{\alpha_k} di \]  

where \( L_{k,j,t} \) is the share of the labor force in country \( k \) in period \( t \) that works in sector \( j \); \( x_{k,j,i,t} \) is the number of machines of type \( i \) used in country \( k \) in sector \( j \) in period \( t \); \( A_{k,j,i,t} \) is a measure of the quality of machine \( i \) in country \( k \).

$k$ in sector $j$ in period $t$; and $\alpha_k \in (0, 1)$ is a Cobb-Douglas distributional parameter. Capital good producers buy labor in a competitive market and take the price of machines as given.

Suppose for simplicity that the number of workers in both economies is identical and constant over time. Then, without loss of generality, the supply of labor can be normalized to one in both countries. In equilibrium, labor demand cannot exceed labor supply:

$$L_{k,c,t} + L_{k,d,t} \leq 1$$

In both countries, both sectors, and every period, each machine is produced by a monopolist at a cost of $\psi > 0$ units of the final good. Assume for simplicity that machines fully depreciate after one period. The monopolist for machine $i$ in sector $j$ in country $k$ chooses the price for that machine in each period.

### 2.3 Scientists and Entrepreneurs

In every period, each scientist chooses to conduct research to improve the quality of machines in either the clean sector or the dirty sector. Each scientist is then randomly allocated to at most one machine in the chosen sector. Because each sector in each country contains infinitely many machines, the probability that two scientists (from the same country or from different countries) are allocated to the same machine approaches zero. Therefore, assume that in any period, machine $i$ in sector $j$ is either produced in the North or the South, but not both.

The probability that a scientist in country $k$ innovates successfully in sector $j$ is $\eta_j \in (0, 1)$. A successful innovation increases the quality of machine $i$ in sector $j$ in country $k$ from $A_{k,j,i,t}$ to $(1 + \gamma_k)A_{k,j,i,t}$, where $\gamma_k > 0$. Each successful scientist receives a one-period patent and becomes the entrepreneur for his or her machine in the current period. For machines that have not been improved, patents are randomly allocated to an entrepreneur.

Normalizing the supply of scientists to one in both countries, the market clearing condition for scientists is given by

$$L_{k,c,t} + L_{k,d,t} \leq 1$$
$s_{k,c,t} + s_{k,d,t} \leq 1 \tag{7}$

Next, define the average productivity in sector $j$ and country $k$ in time $t$ as

$$A_{k,j,t} \equiv \int_{0}^{1} A_{k,j,i,t} \, di \tag{8}$$

It follows that

$$A_{k,j,t} = s_{k,j,t} \int_{0}^{1} [\eta_{k,j}(1 + \gamma_k)A_{k,j,i,t-1} + (1 - \eta_{k,j})A_{k,j,i,t-1}] \, di + (1 - s_{k,j,t}) \int_{0}^{1} A_{k,j,i,t-1} \, di$$

$$= s_{k,j,t}(\eta_{k,j} + \eta_{k,j}\gamma_k + 1 - \eta_{k,j}) \int_{0}^{1} A_{k,j,i,t} \, di + (1 - s_{k,j,t}) \int_{0}^{1} A_{k,j,i,t-1} \, di$$

$$= (\eta_{k,j}s_{k,j,t} + \gamma_k\eta_{k,j}s_{k,j,t} + s_{k,j,t} - \eta_{k,j}s_{k,j,t} + 1 - s_{k,j,t}) \int_{0}^{1} A_{k,j,i,t-1} \, di$$

$$= (1 + \gamma_k\eta_{k,j}s_{k,j,t})A_{k,j,t-1} \tag{9}$$

Finally, assume that the quality of the environment degrades in proportion to the world quantity of the dirty capital good in the current period and regenerates in proportion to its own level in the previous period so that $S_t$ is given by

$$S_t = \begin{cases} 
0 & \quad -\xi(Y_{N,d,t-1} + Y_{S,d,t-1}) + (1 + \delta)S_{t-1} \in (-\infty, 0] \\
-\xi(Y_{N,d,t-1} + Y_{S,d,t-1}) + (1 + \delta)S_{t-1} & \quad -\xi(Y_{N,d,t-1} + Y_{S,d,t-1}) + (1 + \delta)S_{t-1} \in (0, \bar{S}) \\
\bar{S} & \quad -\xi(Y_{N,d,t-1} + Y_{S,d,t-1}) + (1 + \delta)S_{t-1} \in [\bar{S}, \infty)
\end{cases} \tag{10}$$
Or equivalently,

$$S_t = \min \{ \max \{ -\xi(Y_{N,d,t-1} + Y_{S,d,t-1}) + (1 + \delta)S_{t-1}, 0 \}, \bar{S} \}$$

As AABH notes, this law of motion captures the idea that the environment loses its ability to repair itself as it degrades. For example, deforestation reduces the rate at which carbon is absorbed from the atmosphere. Climate scientists, that pollution reduces the environment’s ability to heal itself. Climate scientists are also concerned that the quality of the environment will have a “point of no return;” that is, it can degrade so much that it will never recover. Equation 10 allows for this. If $S_t = 0$ for some finite $t$, then in period $\tau = t + 1$, $(1 + \delta)S_{\tau-1} = 0$ so $S_\tau = 0$. It follows immediately that once the quality of the environment reaches zero, it stays at zero forever. From equation 2, this implies that the utility of the representative household will approach negative infinity in the limit in each period $\tau > t$. For the analysis that follows it will be useful to say that an environmental disaster occurs if $S_t = 0$ for some finite $t$.

## 3 Autarkic Laissez-Faire Equilibrium in the South

In this section, I characterize the laissez-faire equilibrium in the South when there is no trade between the North and South. The results in this section are very similar to those in AABH.

Given $\rho$, $\psi$, $\epsilon_S$, $\alpha_S$, $\gamma_S$, $\eta_{S,c}$, $\eta_{S,d}$, $A_{S,c,0}$, $A_{S,d,0}$, and $Y_{N,d,t-1}$, an equilibrium in the South in period $t$ under autarky is characterized by the price of machines ($p_{S,j,i,t}$), demand for machines ($x_{S,j,i,t}$), price of capital goods ($p_{S,j,t}$), demand for capital goods ($Y_{S,j,t}$), wage ($w_{S,t}$), demand for labor ($L_{S,j,t}$), allocation of scientists ($s_{S,c,t}$), and environmental quality ($S_t$) such that in period $t$: (i) $p_{S,j,i,t}$ maximizes profits for the monopolist that produces machine $i$ in sector $j$; (ii) $x_{S,j,i,t}$ and $L_{S,j,t}$ maximize profits for the producers of the capital good in sector $j$; (iii) $Y_{S,c,t}$ and $Y_{S,d,t}$ maximize profits for the producers of the final good; (iv) $s_{S,c,t}$ and $s_{S,d,t}$ maximize expected profits for a researcher; (v) $w_{S,t}$ and $p_{S,j,t}$ clear the labor and capital goods markets respectively; and (vi) $S_t$ is given by equation 10.

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8 AABH, p. 141-2
If the machines in the dirty sector are more productive than those in the clean sector, and if the difference is large enough, then innovation will begin in the dirty sector. Formally, innovation begins in the dirty sector in country \( k \) if:

\[
\frac{A_{k,c,0}}{A_{k,d,0}} < \min \left\{ \left( \frac{\eta_{k,c}}{\eta_{k,d}} \right)^{\frac{1}{\varphi_k}} (1 + \gamma_k \eta_{k,c})^{\frac{\varphi_k + 1}{\varphi_k}}, \left( \frac{\eta_{k,c}}{\eta_{k,d}} \right)^{\frac{1}{\varphi_k}} (1 + \gamma_k \eta_{k,d})^{\frac{\varphi_k + 1}{\varphi_k}} \right\}
\]

where \( \varphi_k \equiv (1 - \alpha_k)(1 - \epsilon_k) \) to simplify the notation. Following AABH, I will call that condition Assumption 1.

**Lemma 1:** In equilibrium, innovation at time \( t \) will occur in the South in the clean sector and not in the dirty sector when

\[
\eta_{s,c} A_{s,c,t-1}^{-\varphi_S} < \eta_{s,d}(1 + \gamma_S \eta_{s,c})^{\varphi_S + 1} A_{s,d,t-1}^{-\varphi_S},
\]

in the dirty sector and not in the dirty sector when

\[
\eta_{s,d} A_{s,d,t-1}^{-\varphi_S} < \eta_{s,c}(1 + \gamma_S \eta_{s,d})^{\varphi_S + 1} A_{s,c,t-1}^{-\varphi_S},
\]

and in both sectors when

\[
\eta_{s,c}(1 + \gamma_S \eta_{s,d} s_{s,d,t})^{\varphi_S + 1} A_{s,c,t-1}^{-\varphi_S} = \eta_{s,d}(1 + \gamma_S \eta_{s,c} s_{s,c,t})^{\varphi_S + 1} A_{s,d,t-1}^{-\varphi_S}
\]

(with \( s_{s,c,t} + s_{s,d,t} = 1 \)).

**Proof:** See Appendix A. (This is similar to Lemma 1 in AABH.) ☐

Lemma 1 implies that innovation in period \( t \) will occur in the sector that was more productive in period \( t - 1 \) if the capital goods are gross substitutes, (that is, if \( \epsilon_S > 1 \) or equivalently if \( \varphi_S < 0 \)).

**Proposition 1:** If \( \epsilon_S > 1 \) and Assumption 1 is true in the South, then a unique autarkic laissez-faire equilibrium exists in the South. In that equilibrium, innovation occurs in the dirty sector only and the long run growth rate of dirty capital good production is \( \gamma_S \eta_{s,d} \).

**Proof:** See Appendix A. (This is similar to Proposition 1 in AABH.) ☐
It follows from the law of motion for environmental quality (given by equation 10) that for any period $t$ in which total world production of the dirty capital good is greater than $(1 + \delta)\xi^{-1}\bar{S}$, an environmental disaster occurs in period $t + 1$. When the capital goods are substitutes and Assumption 1 is true, Proposition 1 establishes that $Y_{S,d,t}$ grows at a constant positive rate forever. In AABH, where there is only 1 country in the world, it follows that the laissez-faire equilibrium always leads to an environmental disaster in a finite number of periods. The same result is true with two countries in autarky. Without trade, there is no means for the government in the North to influence the choices of producers in the South. In some period, the South will produce enough of the dirty capital good to cause an environmental disaster in the next period, no matter how much the North makes. Even if the North reduces its dirty capital production, the growth rate of dirty capital production in the South will not be affected. Therefore (proof omitted):

**Proposition 2:** If $\epsilon_S > 1$ and Assumption 1 is true in the South, then the autarkic laissez-faire equilibrium in the South necessarily leads to an environmental disaster in a finite number of periods.

For reasonable parameter values, the North cannot prevent or delay an environmental disaster when no trade occurs. Therefore, the welfare of the representative household in the North, given by equation 3, will approach negative infinity. For this reason, the social planner’s problem in the North under autarky is omitted from the analysis.

## 4 Trade

In this section, I show that the North can prevent an environmental disaster when trade in clean and dirty capital goods is permitted, and I identify a policy that does so. The first step is to formally introduce trade to the model described in Section 2.
4.1 The Model with Trade

When trade in capital goods is permitted, the amount of good $j$ that is produced in country $k$ is not necessarily the same as the amount demanded there, even in equilibrium. For the remainder of the analysis, denote the amount of good $j$ produced in country $k$ in period $t$ by $\tilde{Y}_{k,j,t}$. The amount demanded is denoted $\hat{Y}_{k,j,t}$. The trade analog to equation 5 is:

$$\tilde{Y}_{k,j,t} = L_{k,j,t}^{1-\alpha_k} \int_0^1 A_{k,j,i,t}^{1-\alpha_k} x_{k,j,i,t} \, di$$ \hspace{1cm} (11)

Final good producers cannot buy more capital goods than the quantity produced. Mathematically, that means:

$$\tilde{Y}_{N,c,t} + \tilde{Y}_{S,c,t} \geq \hat{Y}_{N,c,t} + \hat{Y}_{S,c,t} \hspace{1cm} (12)$$

$$\tilde{Y}_{N,d,t} + \tilde{Y}_{S,d,t} \geq \hat{Y}_{N,d,t} + \hat{Y}_{S,d,t} \hspace{1cm} (13)$$

For simplicity, assume further that neither country can run a trade deficit, so the world market for capital goods must clear in every period:

$$p_{c,t} \left( \tilde{Y}_{N,c,t} - \hat{Y}_{N,c,t} \right) = p_{d,t} \left( \tilde{Y}_{N,d,t} - \hat{Y}_{N,d,t} \right) \hspace{1cm} (14)$$

$$p_{c,t} \left( \tilde{Y}_{S,c,t} - \hat{Y}_{S,c,t} \right) = p_{d,t} \left( \tilde{Y}_{S,d,t} - \hat{Y}_{S,d,t} \right) \hspace{1cm} (15)$$

Any three equations chosen from among (12), (13), (14), and (15) will be linearly independent. For mathematical convenience, I discard (15).
4.2 Laissez-Faire Equilibrium in the South

Given $\rho$, $\psi$, $\bar{Y}_{N,d,t-1}$, $\alpha_k$, $\epsilon_k$, $\gamma_k$, $\eta_{k,c}$, $\eta_{k,d}$, $A_{k,c,0}$, and $A_{k,d,0}$ for $k \in \{N,S\}$, an equilibrium in the South in period $t$ in which trade in capital goods is permitted is characterized by the price of machines ($p_{S,j,i,t}$), demand for machines ($x_{S,j,i,t}$), world price of capital goods ($p_{j,t}$), demand for capital goods ($\hat{Y}_{S,j,t}$), production level for capital goods ($\tilde{Y}_{S,j,t}$), wage ($w_{S,t}$), demand for labor ($L_{S,j,t}$), allocation of scientists ($s_{S,j,t}$), and environmental quality ($S_t$) such that: (i) $p_{S,j,i,t}$ maximizes profits for the monopolist that produces machine $i$ in sector $j$; (ii) $x_{S,j,i,t}$ and $L_{S,j,t}$ maximize profits for the producers of the capital good in sector $j$; (iii) $\hat{Y}_{S,j,t}$ and $\hat{Y}_{S,d,t}$ maximize profits for the producers of the final good; (iv) $s_{S,c,t}$ and $s_{S,d,t}$ maximize expected profits for a researcher; (v) $w_{S,t}$ and $p_{j,t}$ clear the labor and capital goods markets respectively; and (vi) $S_t$ is given by equation 10.

The profit maximizing choices in the South depend on the Northern demand for imports and supply of exports, so we cannot completely characterize the equilibrium without solving the social planner’s problem in the North. Moreover, the first order conditions for welfare maximization in the North and profit maximization in the South constitute a system of 46 linearly independent equations. Although there is no reason to believe that this system is impossible to solve analytically, it is impractical to do so. Fortunately, a partial solution is sufficient for several results.

**Proposition 3:** In equilibrium, the following are true in the South:
\[ p_{S,j,t} = \frac{\psi}{\alpha_S} \]  

(16)

\[ \frac{p_{c,t}}{p_{d,t}} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\alpha_S} \]  

(17)

\[ \tilde{Y}_{S,c,t} = \left( \frac{\alpha^2 S}{\psi} \right)^{\alpha_S} \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S} + 1 \right]^{-\alpha_S} A_{S,c,t} L_{S,c,t} \]  

(18)

\[ \tilde{Y}_{S,d,t} = \left( \frac{\alpha^2 S}{\psi} \right)^{\alpha_S} \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S} + 1 \right]^{-\alpha_S} A_{S,d,t} L_{S,d,t} \]  

(19)

**Proof:** See Appendix B1. ■

Equation 16 establishes the usual result that the prices of goods sold by monopolistically competitive firms will be a constant markup over marginal cost. Equation 17 supports the intuition that the price of the input that is produced with more productive machines will be cheaper. Further, relative prices increase more slowly than relative productivities.

The relationship between productivities and input production is ambiguous; depending on the way labor demand responds to changes in productivities, capital good production could be increasing or decreasing in \( A_{S,c,t} \) and \( A_{S,d,t} \). This means that without solving completely for equilibrium, we do not know how to manipulate the growth rate of dirty capital production by adjusting the growth rates of the productivities. For example, if we could know that \( \tilde{Y}_{S,d,t} \) were increasing in \( A_{S,d,t} \) but not in \( A_{S,c,t} \) (for relevant parameter values), and that the South were not fully specialized in clean input production, then to prevent an environmental disaster it would be essential not to do any research in the dirty sector in the long run.\(^9\)

\(^9\) See Appendix C for a derivation of the trade flows for which the South only innovates in the clean sector.
4.3 Social Planner’s Allocation in the North

Among all of the allocations that the Social Planner can choose, a proper subset of those does not result in an environmental disaster. The socially optimal allocation is in this subset. If the subset is empty, household welfare approaches negative infinity for every allocation, so none of the allocations are optimal. If there is only one allocation that does not yield an environmental disaster, then it must be the optimal one. If there are multiple allocations that avoid disaster, the necessary and sufficient conditions for Pareto efficiency are as follows.

Suppose that the social planner is active beginning in period $t = 1$. Given $u(\cdot)$, $\rho$, $\psi$, $\tilde{Y}_{S,d,t-1}$ for all $t \in \{1, 2, \ldots\}$, $\alpha_k$, $\epsilon_k$, $\gamma_k$, $\eta_{k,c}$, $A_{k,c,0}$, and $A_{k,d,0}$ for $k \in \{N, S\}$, the socially optimal allocation in the North in which trade in capital goods is permitted is characterized by the demand for machines ($x_{N,j,i,t}$), demand for capital goods ($\tilde{Y}_{N,j,t}$), production level for capital goods ($\bar{Y}_{N,j,t}$), demand for labor ($L_{N,j,t}$), allocation of scientists ($s_{N,j,t}$), environmental quality ($S_t$), productivity of machines ($A_{N,j,i,t}$) demand for final goods ($Y_{N,t}$), and consumption ($C_{N,t}$) in every period $t \geq 1$ that maximize the welfare of the representative household (given by equation 3) subject to the production functions for the final good and the capital goods (given by equations 4 and 11), the demand for laborers and scientists does not exceed the supply (equations 6 and 7), the productivity in sector $j$ in country $k$ evolves according to equation 9, the quality of the environment is given by equation 10, the world demand for clean and dirty inputs is less than or equal to the supply (equations 12 and 13) and trade is balanced in every period (equation 14).

When resources are optimally allocated, marginal changes in the allocation have no effect on welfare. Usually, to find the allocation that maximizes welfare subject to a set of constraints, we would write a Lagrangian and take first order conditions with respect to the variables that the social planner chooses. However, because we required that world input markets clear at the optimum, the Lagrangian would include several variables - $p_{j,t}$, $\tilde{Y}_{S,j,t}$, and $\bar{Y}_{S,j,t}$ - that the social planner cannot choose, but do change in response to the social planner’s other choices. It follows from Proposition 3 and equation 12 that $p_{j,t}$, $\tilde{Y}_{S,j,t}$, and $\bar{Y}_{S,j,t}$ respond to changes in $\tilde{Y}_{N,j,t}$ and $\bar{Y}_{N,j,t}$, but it is not obvious how. It is comparatively straightforward to identify an allocation for which no disaster occurs and to determine the tax and subsidy that produce that allocation.
Proposition 3: Suppose $\epsilon_S > 1$. If the initial quality of the environment is sufficiently high, Northern production of the dirty input is low enough in the long run, and $L_{S,d,t} = 0$ in the long run, then an environmental disaster will not occur. The condition that $L_{S,d,t} = 0$ in equilibrium is equivalent to

$$\tilde{Y}_{N,c,t} - \tilde{Y}_{N,d,t} \leq \frac{\left( \frac{\alpha_S}{\psi} \right)^{\frac{\alpha_S}{1-\alpha_S}} \left[ \left( \frac{(1+\gamma_S)A_{S,c,t}}{A_{S,d,t}} \right)^{-\psi_S} + 1 \right]^{\frac{-\alpha_S}{\psi_S}}}{\left( \frac{(1+\gamma_S)A_{S,c,t}}{A_{S,d,t}} \right)^{-\psi_S} + 1}$$  \hspace{1cm} (20)

Proof: Regardless of the resource allocation in the North, the equilibrium production of the dirty good in the South is given by (20). When $L_{S,d,t} = 0$, then $\tilde{Y}_{S,d,t} = 0$. In that case, environmental damage in period $t$ is proportional to dirty input production in the North. It follows immediately from (11) that the quality of the environment will never reach zero if the initial quality of the environment is sufficiently high and Northern production of the dirty input is low enough in the long run.

For proof that (20) is true if and only if $L_{S,d,t} = 0$, see Appendix B2. ■

Because (20) is sufficient for (12), (13), and (14) when $L_{S,d,t} = 0$, it is now possible to write the Lagrangian for the social planner’s problem. Intuitively, (17) implies that the less productive is the Southern clean sector relative to its dirty sector, the more clean capital the North needs to import (and the more dirty capital the North needs to export) to persuade Southern producers not to make any dirty capital themselves.

Suppose that an environmental disaster will occur unless the South completely specializes in clean production. Then the socially optimal allocation in the North is the Pareto efficient allocation that satisfies equation 20. If disaster can be avoided without $L_{S,d,t} = 0$ in the long run, then among the Northern allocations for which equation 20 is true, we are still most interested in the welfare-maximizing one. This motivates Proposition 4.

Proposition 4: Suppose $\epsilon_N > 1$, $\epsilon_S > 1$, and Assumption 1 is true in the North and the South. Then, among the allocations for which equation 17 holds, there is a unique, welfare maximizing allocation and it can be achieved by imposing a tax on input producers and giving a subsidy to machine producers. The tax, $\tau_{N,t}$ is
\[
\tau_{N,t} = \frac{\xi \sum_{v=t+1}^{\infty} (1+\delta)^{v-t} \partial u(C_{N,v},S_v) \theta S_v}{\delta S_v} I_{S_{t+1},...,S_v<s} (1 + \delta)^{\lambda_{N,d,t}}.
\]

When the social gain from innovation is higher in the clean sector,

\[
\eta_{N,c} (1 + \gamma_N \eta_{N,c} s_{N,c,t})^{-1} \sum_{t=t}^{\infty} \lambda_{N,c,t} L_{N,c,t} p_{N,c,t}^{\alpha N} \bar{A}_{N,c,t} > 1
\]
\[
\eta_{N,d} (1 + \gamma_N \eta_{N,d} s_{N,d,t})^{-1} \sum_{t=t}^{\infty} \lambda_{N,d,t} L_{N,d,t} p_{N,d,t}^{\alpha N} \bar{A}_{N,d,t} > 1
\]

The subsidy, \( q_{N,t} \), that persuades scientists to do research only in the clean sector satisfies

\[
q_{N,t} > \frac{\eta_{N,d}}{\eta_{N,c}} \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-1} \left( \frac{A_{N,c,t}}{A_{N,d,t}} \right)^{\alpha N-1} \left( \frac{A_{N,c,t-1}}{A_{N,d,t-1}} \right)^{-1} \frac{\bar{Y}_{N,c,t}}{\bar{Y}_{N,d,t}} - 1
\]

**Proof:** See Appendix B2. (This is similar to Proposition 5 in AABH.)

## 5 Conclusion

The aims of this paper are to determine whether a policy intervention is necessary to prevent an environmental disaster in a multi-country world and, if it is, to describe the best policies that large economies can pursue alone. There are two principle findings. First, if, in any country with laissez-faire policy, the dirty sector is sufficiently advanced relative to the clean sector and the dirty and clean parts of the production process are substitutes, then an environmental disaster will occur in the absence of trade. Second, as long as the dirty and clean elements of production are substitutes in both countries, one country can unilaterally avert an environmental disaster by imposing a tax on the domestic producers of dirty goods, subsidizing research in the clean sector, and exporting enough of the dirty input that the other country fully specializes in clean production. This sort of unilateral policy is effective even if the dirty sectors in both countries are initially more advanced than the clean sectors.
In this first analysis, I assumed that the South never decides to institute environmental policies. There are several theoretically interesting and practically valuable extensions. The social planner in the North could assume that there is some probability that the South will institute an environmental policy beginning next period, and that this probability grows with time, environmental damage, or per capita wealth. The policy problem could also be considered as a multi-period strategic game in which the North and South decide in each period whether to choose a cooperative environmental policy (preventing environmental damage is cheaper when they cooperate) or to “cheat” by choosing to maximize per-capita consumption, total output growth, or some other self-interested short-term determinant of welfare instead of the socially optimal environmental policy. The economist might be interested in finding the optimal number of punishment periods in response to cheating.

There are least two other extensions of AABH. In Acemoglu, Aghion, and Hemous (2014), the authors show that under slightly different assumptions about the evolution of productivity in the South than the ones adopted in this essay, the North can only prevent an environmental disaster without global policy coordination if the two inputs are substitutes in both countries, no trade occurs, and the South mimics innovations made in the North. The authors focus on the benefits of coordination relative to unilateral action. In Hemous’s (2015 - unpublished) extension of AABH, innovations make it possible to produce “dirty” inputs without damaging the environment. His results are similar to the conclusions in this essay; the North can unilaterally prevent an environmental disaster by incentivizing the South to specialize completely in the clean good. The North then subsidizes research to make the dirty good without polluting.

Further research could also relax one or more of the assumptions adopted here for simplicity. For example, without the assumption of stable population, we could ask whether the economy can sustain positive per-capita growth in consumption indefinitely without causing a disaster. Research could also consider cases in which countries may run trade deficits or build on the research done in other countries.
6 Works Cited


7 Appendix A: Autarkic Laissez-Faire Equilibrium in the South

7.1 Capital Goods Producers

Capital goods producers in sector $j$ choose $x_{S,j,i,t}$ and $L_{S,j,t}$ to maximize profits. The profits for the capital goods producers in the South in sector $j$ in period $t$ are given by:

$$\pi_{S,j,t} = p_{S,j,t}Y_{S,j,t} - w_{S,t}L_{S,j,t} - \int_0^1 p_{S,j,i,t}x_{S,j,i,t}di$$

Use the production function for capital goods, equation 5, to substitute for $Y_{S,j,t}$:

$$\pi_{S,j,t} = p_{S,j,t}L_{S,j,t}^{1-\alpha_S} \int_0^1 A_{S,j,i,t}^{1-\alpha_S}x_{S,j,i,t}^{\alpha_S}di - w_{S,t}L_{S,j,t} - \int_0^1 p_{S,j,i,t}x_{S,j,i,t}di$$

The FONCs are:

$$\frac{\partial \pi_{S,j,t}}{\partial x_{S,j,i,t}} = 0 = p_{S,j,t}L_{S,j,t}^{1-\alpha_S} A_{S,j,i,t}^{1-\alpha_S}x_{S,j,i,t}^{\alpha_S-1} - p_{S,j,i,t}$$

$$\frac{\partial \pi_{S,j,t}}{\partial L_{S,j,t}} = 0 = p_{S,j,t}(1-\alpha_S)L_{S,j,t}^{-\alpha_S} \int_0^1 A_{S,j,i,t}^{1-\alpha_S}x_{S,j,i,t}^{\alpha_S}di - w_{S,t}$$

Solving the FONC with respect to $x_{S,j,i,t}$ for machine demand yields:

$$x_{S,j,i,t} = \left(\frac{\alpha_s p_{S,j,t}}{p_{S,j,i,t}}\right)^{\frac{1}{1-\alpha_S}} A_{S,j,i,t} L_{S,j,t}$$ (21)

Solving the FONC with respect to $L_{S,j,t}$ for the wage:

$$w_{S,t} = p_{S,j,t}(1-\alpha_S)L_{S,j,t}^{-\alpha_S} \int_0^1 A_{S,j,i,t}^{1-\alpha_S}x_{S,j,i,t}^{\alpha_S}di$$ (22)
7.2 Machine Producers

The monopolist that produces machine $i$ in sector $j$ chooses $p_{S,j,i,t}$ to maximize profits. The profits for that monopolist in the South in period $t$ are given by:

$$\pi_{S,j,i,t} = (p_{S,j,i,t} - \psi)x_{S,j,i,t}$$

Use (21) to substitute for $x_{S,j,i,t}$:

$$\pi_{S,j,i,t} = (p_{S,j,i,t} - \psi)\left(\frac{\alpha S p_{S,j,t}}{p_{S,j,i,t}}\right)^{\frac{1}{1-\alpha S}}A_{S,j,i,t}L_{S,j,t}$$

The FONC is:

$$\frac{\partial \pi_{S,j,i,t}}{\partial p_{S,j,i,t}} = 0 = (\alpha S p_{S,j,t})^{\frac{1}{1-\alpha S}}A_{S,j,i,t}L_{S,j,t}p_{S,j,i,t}^{\frac{1}{1-\alpha S}}\left[1 - (p_{S,j,i,t} - \psi)\left(\frac{1}{1-\alpha S}\right)\frac{1}{p_{S,j,i,t}}\right]$$

Solving for $p_{S,j,i,t}$ yields:

$$p_{S,j,i,t} = \frac{\psi}{\alpha S} \quad (23)$$

The price of machines in both sectors is a constant mark-up over marginal cost. Because I assumed that the marginal cost, $\psi$, is the same for both sectors, the price of machines is the same too. Substitute (23) into (21) for $p_{S,j,i,t}$:
\[ x_{S,j,i,t} = \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{1 - \alpha_S}} p_{S,j,t}^{-\alpha_S} A_{S,j,i,t} L_{S,j,t} \quad (24) \]

Substitute (24) into (22) for \( x_{S,j,i,t} \) and substitute (8) in for \( A_{S,j,i,t} \):

\[
w_{S,t} = p_{S,j,t} (1 - \alpha_S) L_{S,j,t}^{-\alpha_S} \int_0^1 A_{S,j,i,t}^{1-\alpha_S} \left[ \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{1 - \alpha_S}} p_{S,j,t}^{-\alpha_S} A_{S,j,i,t} L_{S,j,t} \right]^{\alpha_S} \, di
\]

\[
= \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{\alpha_S}{1 - \alpha_S}} (1 - \alpha_S) p_{S,j,t}^{-\alpha_S} \int_0^1 A_{S,j,i,t} \, di
\]

\[
= \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{\alpha_S}{1 - \alpha_S}} (1 - \alpha_S) p_{S,j,t}^{-\alpha_S} A_{S,j,t}
\]

(25) is true for \( j = c, d \), so we can write:

\[
\frac{p_{S,c,t}}{p_{S,d,t}} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-1 - \alpha_S}
\]

Equation 26 shows that the relative price of the clean intermediate good to the dirty intermediate good is a decreasing function of the average productivity of clean machines relative to the average productivity of dirty machines. That is, as the clean good becomes relatively more productive, it becomes relatively cheaper. However, the relative price does not decrease by as much as the productivity increases. Further, since \( 1 - \alpha_S \in (0, 1) \), the more relatively productive the clean good is, the smaller are the effects from further increases in relative productivity on the relative price. Substitute (24) into (5) for \( x_{S,j,i,t} \):
\[ Y_{S,c,t} = \left( \frac{\alpha_S^2}{\psi} \right) \frac{\alpha_S}{\alpha_S} \frac{\alpha_S}{p_{S,c,t}} A_{S,c,t} L_{S,c,t} \]  
\[ Y_{S,d,t} = \left( \frac{\alpha_S^2}{\psi} \right) \frac{\alpha_S}{\alpha_S} \frac{\alpha_S}{p_{S,d,t}} A_{S,d,t} L_{S,d,t} \]  

(27)  

(28)

7.3 Final Good Producers

The final good producers choose \( Y_{S,c,t} \) and \( Y_{S,d,t} \) to maximize profits. The profits for the final good producers in the South in period \( t \) are given by:

\[ \pi_{S,t} = Y_{S,t} - p_{S,c,t} Y_{S,c,t} - p_{S,d,t} Y_{S,d,t} \]

Use (4) to substitute for \( Y_{S,t} \):

\[ \pi_{S,t} = \left[ \frac{\epsilon_S^g}{\epsilon_S} Y_{S,c,t} + \frac{\epsilon_S^g}{\epsilon_S} Y_{S,d,t} \right] - p_{S,c,t} Y_{S,c,t} - p_{S,d,t} Y_{S,d,t} \]

The FONCs are:

\[ \frac{\partial \pi_{S,t}}{\partial Y_{S,c,t}} = 0 = \frac{\epsilon_S}{\epsilon_S - 1} \left[ \frac{\epsilon_S^g - 1}{\epsilon_S} Y_{S,c,t} + \frac{\epsilon_S^g - 1}{\epsilon_S} Y_{S,d,t} \right] - \frac{1}{\epsilon_S} \left( \frac{\epsilon_S - 1}{\epsilon_S} Y_{S,c,t} - p_{S,c,t} \right) \]

\[ \frac{\partial \pi_{S,t}}{\partial Y_{S,d,t}} = 0 = \frac{\epsilon_S}{\epsilon_S - 1} \left[ \frac{\epsilon_S^g - 1}{\epsilon_S} Y_{S,c,t} + \frac{\epsilon_S^g - 1}{\epsilon_S} Y_{S,d,t} \right] - \frac{1}{\epsilon_S} \left( \frac{\epsilon_S - 1}{\epsilon_S} Y_{S,d,t} - p_{S,d,t} \right) \]
Solving the FONC with respect to $Y_{S,c,t}$ for $p_{S,c,t}$ yields:

$$p_{S,c,t} = \left[ Y_{S,c,t}^\epsilon_S - 1 \right]_{S,c,t} Y_{S,c,t}^{\epsilon_S - 1} Y_{S,c,t}^\epsilon_S - 1$$ (29)

Solving the FONC with respect to $Y_{S,d,t}$ for $p_{S,d,t}$ yields:

$$p_{S,d,t} = \left[ Y_{S,d,t}^\epsilon_S - 1 \right]_{S,d,t} Y_{S,d,t}^{\epsilon_S - 1} Y_{S,d,t}^\epsilon_S - 1$$ (30)

Divide (29) by (30):

$$\frac{Y_{S,c,t}}{Y_{S,d,t}} = \left( \frac{p_{S,c,t}}{p_{S,d,t}} \right)^{-\epsilon_S}$$

Relative demand is a decreasing function of the relative price, as we would expect. If the inputs are gross substitues (that is, if $\epsilon_S > 1$) then a change to relative prices of any given size causes a larger change to relative demand, and the higher are the relative prices, the larger will be the effect of a further increase in relative prices on relative demand.

7.4 Solving for Equilibrium in terms of Productivities

The important equations at this stage are the production functions - (4) and (6) - and (26) through (30). I choose to substitute for prices first. Substitute (29) and (30) into (26) for $p_{S,c,t}$ and $p_{S,d,t}$:

$$\left[ \frac{Y_{S,c,t}^\epsilon_S - 1}{Y_{S,d,t}^\epsilon_S} + \frac{Y_{S,c,t}^\epsilon_S - 1}{Y_{S,d,t}^\epsilon_S} \right]_{S,c,t} Y_{S,c,t}^{\epsilon_S - 1} Y_{S,c,t}^\epsilon_S - 1 = \left[ \frac{Y_{S,c,t}^\epsilon_S - 1}{Y_{S,d,t}^\epsilon_S} + \frac{Y_{S,c,t}^\epsilon_S - 1}{Y_{S,d,t}^\epsilon_S} \right]_{S,d,t} Y_{S,d,t}^{\epsilon_S - 1} Y_{S,d,t}^\epsilon_S - 1$$

Simplifying and solving for $Y_{S,d,t}$:
The relative demand for inputs is an increasing function of their relative productivity. When the inputs are sufficiently substitutable, any given increase in relative productivity causes a larger increase in relative demand. When the inputs are sufficiently substitutable and relative productivity is already high, a further increase in relative productivity will produce a larger increase in relative demand. Substitute (29) into (27) for $p_{S,c,t}$:

$$Y_{S,c,t} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\epsilon_S (1 - \alpha_S)} Y_{S,c,t}$$

(31)

Substitute (30) into (28) for $p_{S,d,t}$:

$$Y_{S,d,t} = \left( \frac{\alpha_S}{\psi} \right)^{\frac{\alpha_S}{1 - \alpha_S}} \left[ Y_{S,c,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} + Y_{S,d,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} \right]^{\frac{-\alpha_S}{\epsilon_S}} Y_{S,c,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} A_{S,c,t} L_{S,c,t}$$

(32)

Substitute (31) into (32) and (33) for $Y_{S,d,t}$ and simplify:

$$Y_{S,c,t} = \left( \frac{\alpha_S}{\psi} \right)^{\frac{\alpha_S}{1 - \alpha_S}} \left[ A_{S,c,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} + A_{S,d,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} \right]^{\frac{-\alpha_S}{\epsilon_S}} A_{S,c,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} A_{S,d,t} L_{S,c,t}$$

(34)

$$Y_{S,c,t} = \left( \frac{\alpha_S}{\psi} \right)^{\frac{\alpha_S}{1 - \alpha_S}} \left[ A_{S,c,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} + A_{S,d,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} \right]^{\frac{-\alpha_S}{\epsilon_S}} \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{1 - \frac{\epsilon_S}{\alpha_S}} A_{S,d,t}^{\frac{\epsilon_S - 1}{\epsilon_S}} L_{S,d,t}$$

(35)

Substitute (8) into (34) for $L_{S,c,t}$:
\[ Y_{S,c,t} = \left( \frac{\alpha_S}{\psi} \right)^{\alpha_S \psi} \left[ A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S} \right]^{-\alpha_S \phi_S} A_{S,d,t}^{\alpha_S} A_{S,c,t} (1 - L_{S,d,t}) \]

Solve for \( L_{S,d,t} \):

\[ L_{S,d,t} = 1 - Y_{S,c,t} \left( \frac{\alpha_S}{\psi} \right)^{\alpha_S \psi} \left[ A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S} \right]^{-\alpha_S \phi_S} A_{S,d,t}^{\alpha_S} A_{S,c,t}^{-1} \]

(36)

Substitute into (35) and solve for \( Y_{S,c,t} \):

\[ Y_{S,c,t} = \left( \frac{\alpha_S}{\psi} \right)^{\alpha_S \psi} \left( A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S} \right)^{-\alpha_S + \phi_S} A_{S,c,t}^{\alpha_S + \phi_S} A_{S,d,t} \]

(37)

Now I begin to solve for the rest of the variables in equilibrium. Substitute (37) into (31) for \( Y_{S,c,t} \):

\[ Y_{S,d,t} = \left( \frac{\alpha_S}{\psi} \right)^{\alpha_S \psi} \left( A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S} \right)^{-\alpha_S + \phi_S} A_{S,c,t}^{\alpha_S + \phi_S} A_{S,d,t} \]

(38)

Substitute (37) into (36) for \( Y_{S,c,t} \):

\[ L_{S,d,t} = \frac{A_{S,c,t}^{\phi_S}}{A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S}} \]

(39)

Substitute (39) into (6) for \( L_{S,d,t} \):

\[ L_{S,c,t} = \frac{A_{S,d,t}^{\phi_S}}{A_{S,c,t}^{\phi_S} + A_{S,d,t}^{\phi_S}} \]

(40)
Substitute (37) and (38) into (29) for $Y_{S,c,t}$ and $Y_{S,d,t}$:

$$p_{S,c,t} = \left( A_{S,c,t}^{\alpha_S + \varphi_S} (\alpha_S)^{\epsilon_S} + (A_{S,c,t}^{\alpha_S} A_{S,d,t}) (\alpha_S)^{\epsilon_S - 1} \right)^{\epsilon_S - 1} A_{S,c,t}^{-\frac{\alpha_S + \varphi_S}{\epsilon_S}}$$

Substitute (37) and (38) into (30) for $Y_{S,c,t}$ and $Y_{S,d,t}$:

$$p_{S,d,t} = \left( A_{S,c,t}^{\alpha_S + \varphi_S} (\alpha_S)^{\epsilon_S} + (A_{S,c,t}^{\alpha_S} A_{S,d,t}) (\alpha_S)^{\epsilon_S - 1} \right)^{\epsilon_S - 1} A_{S,c,t}^{-\frac{\alpha_S + \varphi_S}{\epsilon_S}} A_{S,d,t}^{-\frac{1}{\epsilon_S}}$$

Substitute (37) into (25) for $Y_{S,c,t}$:

$$w_{S,t} = \left( \alpha_S^2 \right)^{1 - \frac{\alpha_S}{\psi}} (1 - \alpha_S) \left( A_{S,c,t}^\varphi + A_{S,d,t}^\varphi \right) \left( A_{S,c,t}^{\alpha_S + \varphi} A_{S,d,t}^{1 - \alpha_S} \right)^{\frac{1}{1 - \alpha_S}}$$

Substitute (37) and (40) into (24) for $Y_{S,c,t}$ and $L_{S,c,t}$:

$$x_{S,c,i,t} = \left( \alpha_S^2 \right)^{1 - \frac{\alpha_S}{\psi}} \left[ \left( A_{S,c,t}^{\alpha_S + \varphi} A_{S,d,t}^{\varphi} \right)^{\epsilon_S - 1} + (A_{S,c,t}^{\alpha_S} A_{S,d,t}) (\alpha_S)^{\epsilon_S - 1} \right]^{\frac{1}{\varphi}} \left( A_{S,c,t}^\varphi + A_{S,d,t}^\varphi \right)$$

Substitute (38) and (39) into (24) for $Y_{S,d,t}$ and $L_{S,d,t}$:

$$x_{S,d,i,t} = \left( \alpha_S^2 \right)^{1 - \frac{\alpha_S}{\psi}} \left[ \left( A_{S,c,t}^{\alpha_S + \varphi} A_{S,d,t}^{\varphi} \right)^{\epsilon_S - 1} + (A_{S,c,t}^{\alpha_S} A_{S,d,t}) (\alpha_S)^{\epsilon_S - 1} \right]^{\frac{1}{\varphi}} \left( A_{S,c,t}^\varphi + A_{S,d,t}^\varphi \right)$$

26
Substitute (37) and (38) into (4) for $Y_{S,c,t}$ and $Y_{S,d,t}$:

$$Y_{S,t} = \frac{\alpha_S^2}{\psi} \left( A^{S,c}_{S,c,t} + A^{S,d}_{S,d,t} \right)^{-\frac{\alpha_S + \varphi_S}{\varphi_S}} \left[ \left( A^{S,c}_{S,c,t} A^{S,d}_{S,d,t} \right)^{\frac{\psi - 1}{\psi}} + \left( A^{S,c}_{S,c,t} A^{S,d}_{S,d,t} \right)^{\frac{\psi - 1}{\psi}} \right]^{\frac{\psi - 1}{\psi - 1}}$$

7.5 Scientists

Scientists maximize expected profits in period $t$ by choosing to do research in either the clean sector or the dirty sector.

$$\pi_{S,j,i,t} = (p_{S,j,i,t} - \psi)x_{S,j,i,t}$$

Use (23) to substitute for $p_{S,j,i,t}$ and (24) for $x_{S,j,i,t}$, we can write:

$$\pi_{S,j,i,t} = \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{\psi - 1}} \frac{1}{p_{S,j,t}^{1 - \frac{\alpha_S}{\varphi_S}}} A_{S,j,i,t} L_{S,j,t}$$

A scientist that chooses to do research in sector $j$ and is allocated to machine $i$ will earn those profits in period $t$ if she innovates successfully. If she does not innovate successfully, her profits will be zero in period $t$. Therefore, expected profits for a scientist that chose sector $j$ AND was allocated to machine $i$, $\Pi_{S,j,i,t}$, are:

$$\Pi_{S,j,i,t} = \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{\psi - 1}} \frac{1}{p_{S,j,t}^{1 - \frac{\alpha_S}{\varphi_S}}} \eta_{S,j}(1 + \gamma_S) A_{S,j,i,t-1} L_{S,j,t}$$

However, the scientist only chooses the sector in which they do research. She is then randomly allocated to a machine within that sector. Therefore, once a scientist has chosen sector $j$, the probability that she is allocated to one machine is the same as the probability that she is allocated to any other machine. There are an uncountably infinite number of machines, so the expected profit for any scientist that chooses sector $j$ is
\( \Pi_{S,j,t} = \int_0^1 \Pi_{S,j,i,t} di \)

\[
= \int_0^1 \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_S^2}{\psi} \right) \frac{1}{1 - \alpha_S} \frac{1}{p_{S,j,t}} \eta_{S,j} (1 + \gamma_S) A_{S,j,t-1} L_{S,j,t} di \\
= \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_S^2}{\psi} \right) \frac{1}{1 - \alpha_S} \frac{1}{p_{S,j,t}} \eta_{S,j} (1 + \gamma_S) L_{S,j,t} \int_0^1 A_{S,j,i,t-1} di \\
= \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_S^2}{\psi} \right) \frac{1}{1 - \alpha_S} \frac{1}{p_{S,j,t}} \eta_{S,j} (1 + \gamma_S) L_{S,j,t} A_{S,j,t-1}
\]

Profits in the clean sector relative to profits in the dirty sector are:

\[
\frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} = \left( \frac{p_{S,c,t}}{p_{S,d,t}} \right) \frac{1}{1 - \alpha_S} \frac{1}{\eta_{S,c} A_{S,c,t-1} L_{S,c,t}} \frac{1}{\eta_{S,d} A_{S,d,t-1} L_{S,d,t}}
\]

Use (23) to substitute for \( \frac{p_{c,t}}{p_{d,t}} \):

\[
\frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} = \left( \frac{A_{S,d,t}}{A_{S,c,t}} \right)^{1 - \alpha_S} \frac{1}{1 - \alpha_S} \frac{1}{\eta_{S,c} A_{S,c,t-1} L_{S,c,t}} \frac{1}{\eta_{S,d} A_{S,d,t-1} L_{S,d,t}}
\]

Use (39) and (40) to substitute for \( L_{S,c,t} \) and \( L_{S,d,t} \):

\[
\frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} = \frac{\eta_{S,c} A_{S,c,t-1}}{\eta_{S,d} A_{S,d,t-1}} \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S - 1}
\]

Substitute for \( A_{S,c,t} \) and \( A_{S,d,t} \) using (9):

\[
\frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} (s_{S,c,t}) = \frac{\eta_{S,c}}{\eta_{S,d}} \left( \frac{1 + \gamma_S \eta_{S,c} s_{S,c,t}}{1 + \gamma_S \eta_{S,d} (1 - s_{S,c,t})} \right)^{-\varphi_S - 1} \left( \frac{A_{S,c,t-1}}{A_{S,d,t-1}} \right)^{-\varphi_S}
\]

28
It is an equilibrium for a scientist to choose sector $j$ if and only if, given the choices of the other scientists, the expected profits in sector $j$ are at least as high as the expected profits in the other sector.

Based on the work already done in AABH:

1. If $-1 - \varphi < 0$, then in equilibrium

$$s_{S,c,t} = \begin{cases} 
1 & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(1) > 1 \\
0 & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(0) < 1 \\
s \in (0, 1) & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(0) > 1 > \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(1) 
\end{cases}$$

2. If $-1 - \varphi > 0$, then in equilibrium

$$s_{S,c,t} = \begin{cases} 
1 & 1 < \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(0) < \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(1) \\
0 & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(0) < \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(1) < 1 \\
1, 0, s \in (0, 1) & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(0) < 1 < \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}}(1) 
\end{cases}$$

3. If $-1 - \varphi = 0$, then in equilibrium

$$s_{S,c,t} = \begin{cases} 
1 & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} > 1 \\
0 & \frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} < 1 
\end{cases}$$

Lemma 1 follows immediately. This concludes the proof of Proposition 1.
8 Appendix B1: Laissez-Faire Equilibrium in the South under Free Trade

8.1 From the Problems for Firms and Scientists

The maximization problems for the capital good producers, monopolists, and final good producers are very similar to the analogous problems from Appendix A. For completeness, I formally state the three problems here anyway.

Capital goods producers in sector $j$ choose $x_{S,j,i,t}$ and $L_{S,j,t}$ to maximize profits. The profits for the capital goods producers in the South in sector $j$ in period $t$ are given by:

$$\pi_{S,j,t} = p_{j,t}L_{S,j,t}^{1-\alpha_S} \int_0^1 A_{S,j,i,t}^{1-\alpha_S}x_{S,j,i,t}^{\alpha_S}di - w_{S,t}L_{S,j,t} - \int_0^1 p_{S,j,i,t}x_{S,j,i,t}di$$

The monopolist that produces machine $i$ in sector $j$ chooses $p_{S,j,i,t}$ to maximize profits. The profits for that monopolist in the South in period $t$ are given by:

$$\pi_{S,j,i,t} = (p_{S,j,i,t} - \psi) \left(\frac{\alpha_S p_{j,t}}{p_{S,j,i,t}}\right)^{\frac{1-\alpha_S}{\alpha_S}} A_{S,j,i,t} L_{S,j,t}$$

The final good producers choose $\hat{Y}_{S,c,t}$ and $\hat{Y}_{S,d,t}$ to maximize profits. The profits for the final good producers in the South in period $t$ are given by:

$$\pi_{S,t} = \left[\hat{Y}_{S,c,t}^{\epsilon_S^{-1}} + \hat{Y}_{S,d,t}^{\epsilon_S^{-1}}\right]^{\frac{\epsilon_S}{\epsilon_S-1}} - p_{c,t}\hat{Y}_{S,c,t} - p_{d,t}\hat{Y}_{S,d,t}$$

From the first order necessary conditions to those three problems, we can derive the following seven equations.
\[ p_{S,j,i,t} = \frac{\psi}{\alpha_S} \]

\[ x_{S,j,i,t} = \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{1-\alpha_S}} \left( p_{j,t}^{1-\alpha_S} A_{S,j,i,t} L_{S,j,t} \right) \]  

(41)

\[ \frac{p_{c,t}}{p_{d,t}} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-(1-\alpha_S)} \]

\[ \tilde{Y}_{S,c,t} = \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{\alpha_S}} p_{c,t}^{\alpha_S} A_{S,c,t} L_{S,c,t} \]  

(42)

\[ \tilde{Y}_{S,d,t} = \left( \frac{\alpha_S^2}{\psi} \right)^{\frac{1}{\alpha_S}} p_{d,t}^{\alpha_S} A_{S,d,t} L_{S,d,t} \]  

(43)

\[ p_{c,t} = \left[ \tilde{Y}_{S,c,t}^{\epsilon_S} \tilde{Y}_{S,c,t}^{\epsilon_S-1} + \tilde{Y}_{S,d,t}^{\epsilon_S} \tilde{Y}_{S,d,t}^{\epsilon_S-1} \right]^{\frac{1}{\epsilon_S-1}} \tilde{Y}_{S,c,t}^{\frac{1}{\epsilon_S-1}} \]  

(44)

\[ p_{d,t} = \left[ \tilde{Y}_{S,c,t}^{\epsilon_S} \tilde{Y}_{S,c,t}^{\epsilon_S-1} + \tilde{Y}_{S,d,t}^{\epsilon_S} \tilde{Y}_{S,d,t}^{\epsilon_S-1} \right]^{\frac{1}{\epsilon_S-1}} \tilde{Y}_{S,d,t}^{\frac{1}{\epsilon_S-1}} \]  

(45)

The analogs to those equations in Appendix A are (23), (24), (26), (27), (28), (29), and (30). Divide (44) by (45):

\[ \frac{\tilde{Y}_{S,c,t}}{\tilde{Y}_{S,d,t}} = \left( \frac{p_{c,t}}{p_{d,t}} \right)^{-\epsilon_S} \]  

(46)

Substitute (17) into (47) for \( \frac{p_{c,t}}{p_{d,t}} \)

\[ \frac{\tilde{Y}_{S,c,t}}{\tilde{Y}_{S,d,t}} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\epsilon_S(1-\alpha_S)} \]  

(47)
Re-arrange (44) and (45):

\[
p_{c,t} = \left[ \frac{\hat{Y}_{S,c,t}}{\hat{Y}_{S,d,t}} \right]^{\frac{1}{\epsilon_S}} + 1
\]

(48)

\[
p_{d,t} = \left[ \frac{\hat{Y}_{S,c,t}}{\hat{Y}_{S,d,t}} \right]^{\frac{1}{\epsilon_S}} + 1
\]

(49)

Substitute (46) into (48) into (49):

\[
p_{c,t} = \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\varphi_S} + 1 \right]^{\frac{1}{\epsilon_S}}
\]

(50)

\[
p_{d,t} = \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S} + 1 \right]^{\frac{1}{\epsilon_S}}
\]

(51)

Substitute (50) into (42) for \( p_{c,t} \) and (51) into (43) for \( p_{d,t} \):

\[
\tilde{Y}_{S,c,t} = \alpha_S^\frac{2}{\psi} \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\varphi_S} + 1 \right]^{\frac{-\alpha_S}{\varphi_S}} A_{S,c,t} L_{S,c,t}
\]

\[
\tilde{Y}_{S,d,t} = \alpha_S^\frac{2}{\psi} \left[ \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S} + 1 \right]^{\frac{-\alpha_S}{\varphi_S}} A_{S,d,t} L_{S,d,t}
\]
As in autarky, scientists in country $k$ maximize expected profits in period $t$ by choosing to do research in either the clean sector or the dirty sector and the expected profits for a scientist that chooses sector $j$ are given by:

$$\Pi_{k,j,t} = \psi \left( \frac{1 - \alpha_k}{\alpha_k} \right) \left( \frac{\alpha_k^2}{\psi} \right)^{1-\alpha_k} \frac{1}{p_{j,t}^1} \eta_{k,j} (1 + \gamma_k) L_{k,j,t} A_{k,j,t-1}$$

(52)

9 Appendix B2: Social Planner’s Allocation in the North under Free Trade

9.1 Determining the constraints

If the South fully specializes in the clean sector, then the South will import dirty capital and export clean capital. From (12), (13), and (46) it is profit-maximizing for Southern final good producers to choose the amount of Southern-made clean capital to keep and the amount of dirty capital to import such that:

$$\tilde{Y}_{S,c,t} - (\hat{Y}_{N,c,t} - \tilde{Y}_{N,c,t}) \geq \left( \frac{p_{c,t}}{p_{d,t}} \right)^{-\epsilon_S} \left( \hat{Y}_{N,d,t} - \tilde{Y}_{N,d,t} \right)$$

Use equation 14 to impose balanced trade:

$$\hat{Y}_{N,c,t} - \tilde{Y}_{N,c,t} \leq \frac{\tilde{Y}_{S,c,t}}{\left( \frac{p_{c,t}}{p_{d,t}} \right)^{1-\epsilon_S} + 1}$$

Use (18) to substitute for $\tilde{Y}_{S,c,t}$:

$$\hat{Y}_{N,c,t} - \tilde{Y}_{N,c,t} \leq \frac{\left( \frac{\alpha_k^2}{\psi} \right)^{1-\epsilon_S} \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\phi_S} \left[ \frac{1}{\epsilon_S} + \frac{A_{S,c,t}}{\phi_S} \right]}{\left( \frac{p_{c,t}}{p_{d,t}} \right)^{1-\epsilon_S} + 1}$$

(53)
Substitute for $\frac{p_{c,t}}{p_{d,t}}$ using (17):

$$\tilde{Y}_{N,c,t} - \tilde{Y}_{N,d,t} \leq \left(\frac{\alpha_2}{\psi}\right)^{1-\alpha_S} \left[ \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{\phi_S} + 1 \right] \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-\phi_S} \frac{-\alpha_S}{\phi_S} A_{S,c,t} \right] \right) + 1 \right)$$

(54)

When $L_{S,d,t} = 0$ and $L_{S,c,t} = 1$;

$$\Pi_{S,c,t} = \psi \left( \frac{1 - \alpha_S}{\alpha_S} \right) \left( \frac{\alpha_2}{\psi} \right)^{1-\alpha_S} \eta_{S,c} (1 + \gamma_S) A_{S,c,t-1} > 0$$

$\Pi_{S,d,t} = 0$

Therefore, in any period in which $L_{S,d,t} = 0$, productivity in the dirty sector does not grow and productivity in the clean sector grows at the rate $\gamma_S \eta_{S,c}$. Equation 20 follows.

9.2 Identifying the optimal policies

Define $\kappa \equiv \left(\frac{\alpha_2}{\psi}\right)^{1-\alpha_S} \left[ \left(\frac{(1 + \gamma_S \eta_{S,c}) A_{S,c,t-1}}{A_{S,d,t-1}}\right)^{\phi_S} + 1 \right] \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-\phi_S} (1 + \gamma_S \eta_{S,c}) A_{S,c,t-1}$.

The Lagrangian is
\[
L = \sum_{t=0}^{\infty} \frac{1}{(1 + \rho)^t} u(C_{N,t}, S_t) + \sum_{t=0}^{\infty} \lambda_{N,t} \left[ Y_{N,t} - \left( \frac{\epsilon_{N,1} \bar{Y}_{N,c,t}}{\epsilon_{N,1} + \epsilon_{N,1} \bar{Y}_{N,d,t}} \right)^{\epsilon_{N,1}} \right] \\
+ \sum_{t=0}^{\infty} \lambda_{N,c,t} \left[ Y_{N,c,t} - L_{N,c,t}^{1-\alpha_N} \int_0^1 A_{N,c,i,t}^{1-\alpha_N} A_{N,c,i,t}^{\alpha_N} \, dx \right] + \sum_{t=0}^{\infty} \lambda_{N,d,t} \left[ Y_{N,d,t} - L_{N,d,t}^{1-\alpha_N} \int_0^1 A_{N,d,i,t}^{1-\alpha_N} A_{N,d,i,t}^{\alpha_N} \, dx \right] \\
+ \sum_{t=0}^{\infty} \phi_{N,L,t} \left[ 1 - L_{N,c,t} - L_{N,d,t} \right] + \sum_{t=0}^{\infty} \phi_{N,s,t} \left[ 1 - s_{N,c,t} - s_{N,d,t} \right] \\
+ \sum_{t=0}^{\infty} \mu_{N,c,t} \left[ A_{N,c,t} - (1 + \gamma_N \eta_N s_{N,c,t}) A_{N,c,t-1} \right] + \sum_{t=0}^{\infty} \mu_{N,d,t} \left[ A_{N,d,t} - (1 + \gamma_N \eta_N s_{N,d,t}) A_{N,d,t-1} \right] \\
+ \sum_{t=0}^{\infty} \omega_t \left[ \min \left\{ \max \left\{ -\xi \bar{Y}_{N,d,t-1} + (1 + \delta) S_{t-1}, 0 \right\}, \bar{S} \right\} - S_t \right] \\
+ \sum_{t=0}^{\infty} \Omega_{N,t} \left[ Y_{N,t} - C_{N,t} - \psi \left( \int_0^1 x_{N,c,i,t} \, dx + \int_0^1 x_{N,d,i,t} \, dx \right) \right] \\
+ \sum_{t=0}^{\infty} \mu_{N,1,t} \left[ \left( \frac{(1 + \gamma_N \eta_N s_{N,c,t}) A_{S,c,t-1}}{A_{S,d,t-1}} \right)^{(1-\alpha_S)} (\bar{Y}_{N,c,t} - \bar{Y}_{N,c,t}) - \bar{Y}_{N,d,t} + \bar{Y}_{N,d,t} \right] \\
+ \sum_{t=0}^{\infty} \mu_{N,2,t} \left[ \bar{Y}_{N,c,t} + \bar{Y}_{N,c,t} - \left( \frac{(1 + \gamma_N \eta_N s_{N,c,t}) A_{S,c,t-1}}{A_{S,d,t-1}} \right)^{\epsilon_S(1-\alpha_S)} (\bar{Y}_{N,d,t} - \bar{Y}_{N,d,t}) \right] 
\]

The FONCs with respect to \( Y_{N,t} \) and \( C_{N,t} \) are:
\[
\frac{\partial L}{\partial Y_{N,t}} = 0 = \lambda_{N,t} + \Omega_{N,t} \\
\frac{\partial L}{\partial C_{N,t}} = 0 = \frac{1}{(1 + \rho)^t} \frac{\partial u(C_{N,t}, S_t)}{\partial C_{N,t}} - \Omega_{N,t}
\]

Combining those yields:

\[
-\lambda_{N,t} = \Omega_{N,t} = \frac{1}{(1 + \rho)^t} \frac{\partial u(C_{N,t}, S_t)}{\partial C_{N,t}}
\] (55)

The FONCs with respect to \(\hat{Y}_{N,c,t}, \hat{Y}_{N,d,t}, \tilde{Y}_{N,c,t},\) and \(\tilde{Y}_{N,d,t}\) are:

\[
\frac{\partial L}{\partial \hat{Y}_{N,c,t}} = 0 = -\lambda_{N,t} \frac{\epsilon_N}{\epsilon_N - 1} \left( \hat{Y}_{N,c,t}^{\epsilon_N^{-1}} + \hat{Y}_{N,d,t}^{\epsilon_N^{-1}} \right) - \left( 1 + \gamma S \eta S_{c,c} \right) \frac{A_{S,c,t}}{A_{S,d,t-1}} - \mu_{N,1,t} - \mu_{N,2,t}
\] (56)

\[
\frac{\partial L}{\partial \hat{Y}_{N,d,t}} = 0 = -\lambda_{N,t} \frac{\epsilon_N}{\epsilon_N - 1} \left( \hat{Y}_{N,c,t}^{\epsilon_N^{-1}} + \hat{Y}_{N,d,t}^{\epsilon_N^{-1}} \right) - \left( 1 + \gamma S \eta S_{c,c} \right) \frac{A_{S,c,t}}{A_{S,d,t-1}} + \mu_{N,1,t} + \mu_{N,2,t}
\] (57)

\[
\frac{\partial L}{\partial \tilde{Y}_{N,c,t}} = 0 = \lambda_{N,c,t} + \mu_{N,1,t} \left( 1 + \gamma S \eta S_{c,c} \right) \frac{A_{S,c,t-1}}{A_{S,d,t-1}} - \mu_{N,2,t}
\] (58)

\[
\frac{\partial L}{\partial \tilde{Y}_{N,d,t}} = 0 = \lambda_{N,d,t} - \omega_{t+1} + \mu_{N,1,t} - \mu_{N,2,t} \left( 1 + \gamma S \eta S_{c,c} \right) \frac{A_{S,c,t-1}}{A_{S,d,t-1}}
\] (59)
Use (58) to substitute for $\mu_{N,2,t}$ in (56), (57), and (59), then use one of the resulting equations to substitute for $\mu_{N,1,t}$.

Defining a proportional tax, $\tau_{N,t} \equiv \frac{\omega_{t+1}^t}{\lambda_{N,d,t}^t}$, and the shadow price of capital good $j$, $p^*_{N,j,t} = \frac{\lambda_{N,j,t}^t}{\lambda_{N,t}^t}$, we can write:

\[
p^*_{N,c,t} = \left( \frac{\hat{Y}_{N,c,t}^{\epsilon N} + \hat{Y}_{N,d,t}^{\epsilon N}}{\hat{Y}_{N,c,t}^{\epsilon N}} \right)^{1/\epsilon N} \hat{Y}_{N,c,t}^{\epsilon N}
\]

\[
p^*_{N,d,t} = \frac{\left( \frac{\hat{Y}_{N,c,t}^{\epsilon N} + \hat{Y}_{N,d,t}^{\epsilon N}}{\hat{Y}_{N,c,t}^{\epsilon N}} \right)^{1/\epsilon N}}{1 - \tau_{N,t}} \hat{Y}_{N,d,t}^{\epsilon N}
\]

Dividing (60) by (61):

\[
\frac{p^*_{N,c,t}}{p^*_{N,d,t}} = \left( \frac{\hat{Y}_{N,c,t}^{\epsilon N}}{\hat{Y}_{N,d,t}^{\epsilon N}} \right) (1 + \tau_{N,t})
\]

The FONC with respect to $S_t$ is:

\[
\frac{\partial L}{\partial S_t} = 0 = \frac{1}{(1 + \rho)^t} \frac{\partial u(C_{N,t}, S_t)}{\partial S_t} - \omega_t + \omega_{t+1}(1 + \delta)I_{S_t < \bar{S}}
\]

where $I_{S_t < \bar{S}}$ is equal to 1 if $S_t < \bar{S}$ and 0 otherwise.

Solving for $\omega_t$:

\[
\omega_t = \sum_{v=t}^{\infty} (1 + \delta)^{v-t} \frac{1}{(1 + \rho)^v} \frac{\partial u(C_{N,v}, S_v)}{\partial S_v} I_{S_t,...,S_v < \bar{S}}
\]

where $I_{S_t,...,S_v < \bar{S}}$ is equal to 1 if $S_t,...,S_v < \bar{S}$ and 0 otherwise.

The FONCs with respect to $s_{N,c,t}$ and $s_{N,d,t}$ are:
\[
\frac{\partial L}{\partial s_{N,c,t}} = 0 = -\phi_{N,s,t} - \mu_{N,c,t} \gamma_N \eta_{N,c} A_{N,c,t-1}
\]
\[
\frac{\partial L}{\partial s_{N,d,t}} = 0 = -\phi_{N,s,t} - \mu_{N,d,t} \gamma_N \eta_{N,d} A_{N,d,t-1}
\]

Setting those equal:

\[
\mu_{N,c,t} \eta_{N,c} A_{N,c,t-1} = \mu_{N,d,t} \eta_{N,d} A_{N,d,t-1}
\]

Finally, the FONCs with respect to \(A_{N,c,i,t}\) and \(A_{N,d,i,t}\) are:

\[
\frac{\partial L}{\partial A_{N,c,i,t}} = 0 = -\lambda_{N,c,t} L_{N,c,t}^{1-\alpha_N} (1 - \alpha_N) A_{N,c,i,t}^{\alpha_N} x_{N,c,i,t}^{\alpha_N} + \mu_{N,c,t} - \mu_{N,c,t+1} (1 + \gamma_N \eta_{N,c} s_{N,c,t+1})
\]
\[
\frac{\partial L}{\partial A_{N,d,i,t}} = 0 = -\lambda_{N,d,t} L_{N,d,t}^{1-\alpha_N} (1 - \alpha_N) A_{N,d,i,t}^{\alpha_N} x_{N,d,i,t}^{\alpha_N} + \mu_{N,d,t} - \mu_{N,d,t+1} (1 + \gamma_N \eta_{N,d} s_{N,d,t+1})
\]

Re-arranging:

\[
\mu_{N,j,t} = \lambda_{N,j,t} L_{N,j,t}^{1-\alpha_N} (1 - \alpha_N) A_{N,j,i,t}^{\alpha_N} x_{N,j,i,t}^{\alpha_N} + \mu_{N,j,t+1} (1 + \gamma_N \eta_{N,j} s_{N,j,t+1})
\]

Substituting in for machine demand:
\[ \mu_{N,j,t} = \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} + \mu_{N,j,t+1} (1 + \gamma_N \eta_{N,j} s_{N,j,t+1}) \]

Adding one more period:

\[ \mu_{N,j,t} = \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} \]

\[ + \left( \lambda_{N,j,t+1} L_{N,j,t+1} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t+1}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} + \mu_{N,j,t+2} (1 + \gamma_N \eta_{N,j} s_{N,j,t+2}) \right) (1 + \gamma_N \eta_{N,j} s_{N,j,t+1}) \]

Using (9):

\[ A_{N,j,t} \mu_{N,j,t} = \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t} \]

\[ + \left( \lambda_{N,j,t+1} L_{N,j,t+1} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t+1}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} + \mu_{N,j,t+2} (1 + \gamma_N \eta_{N,j} s_{N,j,t+2}) \right) A_{N,j,t+1} \]

\[ = \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t} \]

\[ + \lambda_{N,j,t+1} L_{N,j,t+1} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t+1}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t+1} + \mu_{N,j,t+2} A_{N,j,t+2} \]
Solving recursively:

\[
\mu_{N,j,t} = A_{N,j,t}^{-1} \sum_{\tau=t}^{\infty} \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t} \\
= \frac{(1 + \gamma_N \eta_{N,j} s_{N,j,t})^{-1} \sum_{\tau=t}^{\infty} \lambda_{N,j,t} L_{N,j,t} (1 - \alpha_N) \left( \frac{\alpha_N}{\psi} p_{N,j,t}^* \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t}}{A_{N,j,t-1}}
\]

Plugging (64) into (63):

\[
\eta_{N,c} (1 + \gamma_N \eta_{N,c} s_{N,c,t})^{-1} \sum_{\tau=t}^{\infty} \lambda_{N,c,t} L_{N,c,t} p_{N,c,t}^* \left( \frac{\alpha_N}{\psi} A_{N,c,t} \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,c,t} = 1
\]

\[
\eta_{N,d} (1 + \gamma_N \eta_{N,d} s_{N,d,t})^{-1} \sum_{\tau=t}^{\infty} \lambda_{N,d,t} L_{N,d,t} p_{N,d,t}^* \left( \frac{\alpha_N}{\psi} A_{N,d,t} \right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,d,t} = 1
\]

Using (52), the optimal allocation of scientists can be achieved by implementing a proportional subsidy to profits in the clean sector if the North does not fully specialize in dirty capital production:

\[
q_{N,t} > \left( \frac{p_{c,t}}{p_{d,t}} \right)^{\frac{1}{1-\alpha_N}} \eta_{N,d} L_{N,d,t} A_{N,d,t-1} L_{N,c,t} A_{N,c,t-1} - 1
\]

Use (17) to substitute for \( \frac{p_{c,t}}{p_{d,t}} \):

\[
q_{N,t} > \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-1} \eta_{N,d} L_{N,d,t} A_{N,d,t-1} L_{N,c,t} A_{N,c,t-1} - 1
\]

The FONCs with respect to \( x_{N,c,i,t} \) and \( x_{N,d,i,t} \) are:
\[
\frac{\partial L}{\partial x_{N,c,i,t}} = 0 = -\lambda_{N,c,t}L_{N,c,i,t}^{1-\alpha}A_{N,c,i,t}^{1-\alpha}\alpha_N x_{N,c,i,t}^{\alpha_N-1} - \Omega_{N,t}\psi
\]
\[
\frac{\partial L}{\partial x_{N,d,i,t}} = 0 = -\lambda_{N,d,t}L_{N,d,i,t}^{1-\alpha}A_{N,d,i,t}^{1-\alpha}\alpha_N x_{N,d,i,t}^{\alpha_N-1} - \Omega_{N,t}\psi
\]

Use (55) to substitute for \(\Omega_{N,t}\) and solving for machine demand:

\[
x_{N,c,i,t} = \left(\frac{\alpha_N}{\psi} p^*_{N,c,t}\right)^{\frac{1}{1-\alpha_N}} A_{N,c,i,t} L_{N,c,t}
\]
\[
x_{N,d,i,t} = \left(\frac{\alpha_N}{\psi} p^*_{N,d,t}\right)^{\frac{1}{1-\alpha_N}} A_{N,d,i,t} L_{N,d,t}
\]

Substitute machine demand into (5):

\[
\tilde{Y}_{N,j,t} = \left(\frac{\alpha_N}{\psi} p^*_{N,j,t}\right)^{\frac{\alpha_N}{1-\alpha_N}} A_{N,j,t} L_{N,j,t}
\]  
(67)

Use (67) to substitute for the relative demand for labor in (66):

\[
q_{N,t} > \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-1} \eta_{N,d} \frac{\tilde{Y}_{N,c,t}}{\eta_{N,c} Y_{N,d,t}} \left(\frac{p^*_{N,c,t}}{p^*_{N,d,t}}\right)^{\frac{-\alpha}{1-\alpha}} \left(\frac{A_{N,c,t}}{A_{N,d,t}}\right)^{-1} \frac{A_{N,d,t-1}}{A_{N,c,t-1}} - 1
\]  
(68)

The FONCs with respect to \(L_{N,c,t}\) and \(L_{N,d,t}\) are:
\[
\frac{\partial L}{\partial L_{N,c,t}} = 0 = -\lambda_{N,c,t}(1 - \alpha_N) L_{N,c,t}^{-\alpha_N} \int_0^1 A_{N,c,t}^{1-\alpha_N} x_{N,c,t}^{\alpha_N} di - \phi_{N,L,t}
\]

\[
\frac{\partial L}{\partial L_{N,d,t}} = 0 = -\lambda_{N,d,t}(1 - \alpha_N) L_{N,d,t}^{-\alpha_N} \int_0^1 A_{N,d,t}^{1-\alpha_N} x_{N,d,t}^{\alpha_N} di - \phi_{N,L,t}
\]

Setting those equal and substituting in for machine demand:

\[
\frac{p^*_{N,c,t}}{p^*_{N,d,t}} = \left( \frac{A_{N,d,t}}{A_{N,c,t}} \right)^{1-\alpha_N} \tag{69}
\]

Substitute (69) into (68) for \( \frac{p^*_{N,c,t}}{p^*_{N,d,t}} \):

\[
q_{N,t} > \eta_{N,d} \eta_{N,c} \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-1} \left( \frac{A_{N,c,t}}{A_{N,d,t}} \right)^{\alpha_N - 1} \frac{A_{N,c,t}^{1-\alpha_N}}{A_{N,d,t}^{1-\alpha_N}} \frac{\tilde{Y}_{N,c,t}}{\tilde{Y}_{N,d,t}} - 1 \tag{70}
\]

This concludes the proof of Proposition 4.

\section{Appendix C}

Divide (18) by (19) and solve for relative demand for labor:

\[
\frac{\tilde{Y}_{S,c,t}}{\tilde{Y}_{S,d,t}} = \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{-\varphi_S} \left( \frac{A_{S,c,t}^{\varphi_S}}{A_{S,d,t}^{\varphi_S}} + 1 \right)^{-\frac{\alpha_S}{\varphi_S}} A_{S,c,t} L_{S,c,t} A_{S,d,t} L_{S,d,t} \tag{71}
\]

Use (52) to write an expression for relative expected profits:
\[
\frac{\Pi_{S,c,t}}{\Pi_{S,d,t}} = \left( \frac{p_{c,t}}{p_{d,t}} \right)^{\frac{1}{1-\alpha_S}} \frac{L_{S,c,t} A_{S,c,t-1}}{L_{S,d,t} A_{S,d,t-1}}
\]  
\[(72)\]

Substitute (47) into (12) for \(\hat{Y}_{S,c,t}\):

\[
\bar{Y}_{N,c,t} + \bar{Y}_{S,c,t} \geq \hat{Y}_{N,c,t} + \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\epsilon_2(1-\alpha_S)} \hat{Y}_{S,d,t}
\]  
\[(73)\]

Substitute (13) into that for \(\bar{Y}_{S,d,t}\) and solve for \(\tilde{Y}_{S,c,t}\):

\[
\bar{Y}_{S,c,t} \geq \bar{Y}_{N,c,t} + \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\epsilon_2(1-\alpha_S)} \left( \bar{Y}_{N,d,t} + \bar{Y}_{S,d,t} - \hat{Y}_{N,d,t} \right) - \bar{Y}_{N,c,t}
\]  
\[(74)\]

Substitute (74) into (71) for \(\bar{Y}_{S,c,t}\):

\[
\frac{\bar{Y}_{N,c,t} + \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\epsilon_2(1-\alpha_S)} \left( \bar{Y}_{N,d,t} + \bar{Y}_{S,d,t} - \hat{Y}_{N,d,t} \right) - \bar{Y}_{N,c,t}}{\bar{Y}_{S,d,t}} = \left( \frac{\frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S} + 1}{\frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S} + 1} \right)^{-\alpha_S} \frac{A_{S,c,t} L_{S,c,t}}{A_{S,d,t} L_{S,d,t}}
\]  
\[(75)\]

Substitute (18) into (75) for \(\bar{Y}_{S,d,t}\):

\[
\frac{\bar{Y}_{N,c,t} + \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\epsilon_2(1-\alpha_S)} \left( \bar{Y}_{N,d,t} + \left( \frac{A_{S,c,t}}{A_{S,d,t}} \right)^{\varphi_S} + 1 \right)^{-\alpha_S} \frac{A_{S,d,t} L_{S,d,t} - \hat{Y}_{N,d,t}}{A_{S,d,t} L_{S,d,t}} - \bar{Y}_{N,c,t}}{\left( \frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S} + 1 \right)^{-\alpha_S} \frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S}} = \left( \frac{\frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S} + 1}{\frac{A_{S,c,t}}{A_{S,d,t}}^{\varphi_S} + 1} \right)^{-\alpha_S} \frac{A_{S,c,t} L_{S,c,t}}{A_{S,d,t} L_{S,d,t}}
\]  
\[(76)\]

Substitute (6) into (76) for \(L_{S,c,t}\):

43
\[
\begin{align*}
\hat{Y}_{N,c,t} + \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{\epsilon_S(1-\alpha_S)} & \left(\hat{Y}_{N,d,t} + \left(\frac{\alpha_S^2}{\varphi_S}\right) \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-\varphi_S} + 1\right) \frac{\alpha_S}{\varphi_S} A_{S,d,t} L_{S,d,t} - \hat{Y}_{N,d,t}\right) - \hat{Y}_{N,c,t} \\
& = \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{\varphi_S} + 1 \frac{A_{S,c,t} - L_{S,d,t}}{A_{S,d,t}} L_{S,d,t}
\end{align*}
\]

(77)

Solve for \(L_{S,d,t}\), plug the result into (6) to solve for \(L_{S,c,t}\), and substitute for labor demand in both sectors in equation 73. If \(\Pi_{S,c,t} > \Pi_{S,d,t}\), then it is an equilibrium in period \(t\) for scientists in the South to do research in the clean sector only. In that period, productivity in the dirty sector will not grow, and productivity in the clean sector will increase by a factor of \(\gamma_S\eta_{S,c}\). Impose the condition that \(\Pi_{S,c,t} > \Pi_{S,d,t}\), use equation 14 to substitute for \(\tilde{Y}_{N,d,t} - \hat{Y}_{N,d,t}\), and use equation (18) to substitute for relative prices:

\[
\begin{align*}
\hat{Y}_{N,c,t} - \tilde{Y}_{N,c,t} & > \left(\frac{\alpha_S^2}{\varphi_S}\right) \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-\varphi_S} + 1 \frac{\alpha_S}{\varphi_S} A_{S,c,t-1} \left(1 + \gamma_S\eta_{S,c} - \left(\frac{A_{S,c,t}}{A_{S,d,t}}\right)^{-\epsilon_S}\right)
\end{align*}
\]

(78)

Because \(\Pi_{S,c,t} > \Pi_{S,d,t}\), \(A_{S,c,t} = (1 + \gamma_S\eta_{S,c})A_{S,c,t-1}\) and \(A_{S,d,t} = A_{S,d,t-1}\). Thus we can write:

\[
\begin{align*}
\hat{Y}_{N,c,t} - \tilde{Y}_{N,c,t} & > \left(\frac{\alpha_S^2}{\varphi_S}\right) \left(1 + \gamma_S\eta_{S,c}\right) \left(\frac{A_{S,c,t-1}}{A_{S,d,t-1}}\right) \epsilon_S + 1 \frac{\alpha_S}{\varphi_S} \left(1 + \gamma_S\eta_{S,c}\right) A_{S,c,t-1} \left(1 + \gamma_S\eta_{S,c} - (1 + \gamma_S\eta_{S,c})^{-\epsilon_S}\left(\frac{A_{S,c,t-1}}{A_{S,d,t-1}}\right)^{-\epsilon_S}\right)
\end{align*}
\]

(79)