Optimal Mechanism Design for Procurement Auctions for Public Works in Mumbai

Shalini Singh
Northwestern University
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Abstract:

We examine procurement auctions for public works in Mumbai with the goal of understanding game theoretical values and optimal auction structures to recommend to policy makers in this context. We first convert an independent private values framework for traditional auctions to an analogous framework for procurement auctions. Then we use a fixed effects regression to estimate the idiosyncratic components of bids, which we use to estimate bidders’ private value distribution. Finally we then estimate the optimal auction reservation values. Since today these auctions do not include reservation values, we conclude that using these optimal reservation values would likely reduce expected costs for public works tenders in Mumbai.
Introduction

Auctions form an important market institution of today’s global economy and are frequently used to conduct sales of unique objects that do not have a developed market. The extensive use of auctions as mechanisms for allocating resources in such nonmarket environments can be attributed to their relative simplicity and efficiency. Auction rules are easy to understand for potential bidders, and with the right mechanism design, auctions provide an efficient means of maximizing trade surplus.

Procurement auctions are “reverse auctions” in the sense that instead of a single seller selling a good to one of multiple buyers, there is one buyer buying a good or service from one of multiple sellers. They are often used by governments to purchase services of public utility from private parties. Procurement auctions are considered to be a largely efficient way of creating a market for goods which are otherwise difficult to price reasonably. However, because of their design, they are often associated with adverse selection. The bidder with the low valuation wins, and the low valuation may translate into inferior work quality executing the contract, as highlighted by Myerson (1981). The actual auction design varies from case to case and is usually a First-Price Sealed Bid auction (“FPSB”) or a Second-Price Sealed Bid auction (“SPSB”). Our data set is from FPSB so we will focus on this mechanism.

The importance of procurement auctions for executing government policy raises an important question: how can we optimally design auctions? Which auction format (FPSB, SPSB, Dutch, English) maximizes expected revenue and efficiency? Moreover, what does optimality mean for a government? Does it mean maximizing expected revenue or maximizing social surplus?

In this paper, we will use data from FPSB online procurement auctions conducted by the Municipal Corporation of Greater Mumbai (“MCGM”) for various improvement jobs for city roads. After structurally estimating the unobservable parameters (i.e. latent distribution of bidder valuations) and describing the game, we study the auction design and make recommendations from a policy perspective. We utilize nonparametric methods and auction theory adopted for procurement auctions to estimate the
distributions of bidder values and the resulting optimal reservation values in a Mumbai public works procurement auction. This paper is organized into 6 sections: Section 2 surveys existing literature on auctions with emphasis on structural estimation and auction design, Section 3 describes the theory from key publications driving our analysis and develops a framework for our empirical model, Section 4 provides a description of our data in context of the theoretical framework, Section 5 reports results of the empirical analysis and a discussion of results and our analysis, and Section 6 concludes with final recommendations and areas for further research.

1. Literature Review

Since the development of the concept of Bayesian Nash Equilibrium, there has been exhaustive research on auctions – both theoretical and empirical. Auction data (both experimental and field) makes it possible to test theoretical restrictions imposed by game theoretic models and make recommendations policy recommendations.

There are several surveys of empirical and theoretical work on auction data sets by economists. These include a survey of empirical analysis by Hendricks and Porter (“Handbook of Industrial Organization”) with focus on the connection between theory, empirical practice, and public policy. As highlighted by Hendricks and Porter, empirical work in auctions has positive and normative goals.

Positive analysis entails a descriptive analysis of auction data to study the behavior of agents in a game theoretic framework. Specifically, since bids are realizations of bidding strategies which in turn, are a mapping from bidders’ private information, the ex-post distribution of observed bids is a mapping of the underlying distribution of private values of bidders. Thus, one can use this distribution of bidders’ signals to study the relationship between the predictions from theory (for instance Bayesian Nash Equilibrium) and auction data, analyze the assumptions of standard auction theory like risk neutrality of bidders and correlation between private values, and make policy recommendations to enhance efficiency of these
mechanisms in facilitating allocation of resources in the private and public sectors. We will adopt a similar framework for the purposes of this paper.

Other surveys of empirical work on auction data sets have been conducted by Hendricks and Paarsch (1995), Klemperer (2004), and Krishna (2002) have conducted surveys of literature with special emphasis on public policy implications, in addition to the works of McAfee and McMillan (1987). In addition to field data from auction data sets, economists have used experimental data from laboratory auctions. These datasets differ from traditional auction datasets because the experimenter is able to actually observe the signals of private values received, but cannot observe the preferences of bidders. However, analysis of data from actual auctions is more relevant for the purposes of this paper since it provides valuable insights for studying our data from government procurement auctions for highway contracts.

In addition to surveys on empirical work in auction theory, there are summaries on theoretical work in the field. Laffont (1995) provides an overview of literature on pre-game theoretic competitive bidding based on decision theory and distribution-neutral predictions of the three major game theoretic models of independent private values, symmetric common values and asymmetric common values. He concludes that game theoretic models impose significant restrictions on auction data and highlights the need to develop an econometric approach to identification of these models from available data. Indeed, most bidding environments have elements of both private and common value, thereby meriting further development of the Milgrom-Webber affiliated values model.

Milgrom and Weber (1982) present a general symmetric information model for risk-neutral bidders under the primary assumption that the bidders’ valuations are affiliated, i.e. a high value of one bidder’s estimates makes high values of the others’ estimates more likely. This general model can be used to explain the independent private values model and the common values model, as well as a range of intermediate models. Such models can be applied to explain auctions observed in real life since bidders’ valuations tend to have a common component (observed by all bidders), in addition to a private
component (observed only by the bidder). They find that the Dutch and first-price auctions are strategically equivalent in the general model (as in the case of independent private values model). However, the equivalence of English and second-price auctions breaks down when bidders are unsure of their value estimates and the former leads to higher expected prices in general in such cases. In the case of statistically dependent value estimates of bidders, the revenue equivalence theorem (studied by Myerson (1981) and Harris and Raviv (1981)) does not hold since the second-price auction generates higher expected prices than first-price auctions. Milgrom and Weber propose that this prediction explains the predominance of auctions conducted on an ascending-bid basis (as in the case of English auctions). They further extend the general values model to include reserve prices and entry fees and conclude that under certain reasonable assumptions, it is advisable to raise entry fees and reduce reserve price for a fixed screening level to increase expected revenue from the auction. (Milgrom and Weber, 1095).

Laffont and Vuong (1996) recognize the importance of structurally modeling the Bayesian Nash equilibria in game-theoretic models of auctions and present an econometric perspective on structural estimation of games in auction data. Their analysis is based on the behavioral assumption that the observed bids are equilibrium bids of the auction under consideration. Thus, the equilibrium strategy of the game implicit in the auction can be characterized as a structural econometric model in which bids are an increasing function of bidders’ values. Note, however, that the bids are not assumed to be consistent with game theoretic prediction; Laffont and Vuong develop a methodology to test this consistency by studying the monotonicity of the mapping between the distribution of observed bids and that of underlying bidder signals (consisting of both private and common value components as in the Milgrom – Weber affiliated values model). An important contribution of this paper is the analysis on identification of auctions with different message spaces. The authors find that a symmetric affiliated values model is in general unidentified, but a symmetric affiliated private values model (of which the independent private values model is a special case) is identified. Laffont and Vuong (1996) also discuss direct and indirect methods for structural estimation of these models, some of which we discuss in subsequent paragraphs.
The primary goal of structural estimation procedures is to form estimates of components of the underlying game theoretic model. As Porter and Hendricks point out, estimation of unobserved factors like distribution of bidders’ signals and preferences has both positive and normative goals.

The positive goal of such estimation is to describe behavior of players in the context of industrial organization frameworks like collusion versus competition. Porter and Zona (1992) study the possibility of bid rigging in state highway auctions in Long Island in 1980s by comparing the bids submitted by cartel and non-cartel firms. They provide a basic econometric model for estimating the correlation between job-specific and firm-specific factors and the actual bid submitted by the firm and find that the bids submitted by ring members were fundamentally different than those submitted by non-ring members. The latter were related to firm-specific cost measures like backlog of projects, whereas the cartel bids and the corresponding rank distribution did not exhibit such a relationship. The detection of bid-rigging is possible for their dataset since ring meetings were held before the auction to designate a serious bidder and appoint other ring members to submit phony bids, perhaps to create the impression of competition. In cases with only one serious bidder, the seemingly different bidding behavior of firms might be due to idiosyncratic cost differences, which challenges the hypothesis of collusion.

The normative goal of structural estimation is to provide recommendations for mechanism design to enhance expected government revenues or minimize costs. As discussed in subsequent paragraphs, Riley and Samuelson (1981) and Myerson (1981) provide an estimate for the optimal reservation value for auctions under the independent private values paradigm. Both these papers provide valuable insights for optimal mechanism design and will be particularly important for the purposes of this paper. Paarsch (1997) and Haile and Tamer (2003) also estimate the optimal reservation price for such auctions. Computation of optimal reserve values for auctions in the common values environment is more difficult since the underlying distribution of bidders’ signals cannot be estimated by bid data alone.
Significant literature has been devoted to the comparison of various auction formats in terms of expected revenue generated, efficiency, and robustness. Vickrey (1961) proved the four standard auction formats – FPSB, SPSB, English, and Dutch - yield equal expected revenues under the assumptions of bidder risk-neutrality and independent and identical distribution of private values of bidders. Myerson (1981) treats the auction design problem as a decision-making problem under uncertainty of the seller about buyers’ estimates. Thus, it is not possible to design an auction that will guarantee full realization of the object’s value to the seller, but there is a class of optimal auctions that generate highest expected revenue. A mathematical analysis of this decision-making problem leads to the result that Vickrey’s revenue equivalence theorem across auction formats can be extended to the more general subset of auctions which award the good to the bidder with the highest valuation and offers zero expected payoff to the bidder with the lowest value estimate.

Riley and Samuelson (1981) study the family of auctions with these characteristics and the existence of a common equilibrium bidding strategy in which each buyer makes a bid that is a strictly increasing function of his value estimate. They find that under the assumption of risk-neutrality and independent and identical private values, the expected gain of the seller is maximized when the reservation value of the auction is equal to some value strictly greater than the valuation of the object to the seller. An interesting implication of this prediction is that the optimal reserve value below which it is not worthwhile bidding is independent of the number of buyers, and hence, the degree of competition is not predicted to have a direct impact on the optimal auction design, although it does influence equilibrium bidding strategies of bidders under various auction settings. They also study the expected revenue generated by the various forms of auctions under the assumption of risk aversion of buyers (as opposed to the assumption of bidder risk-neutrality in earlier works) and find that bidders make uniformly higher bids in the first-price auctions as they become more risk averse and continue to bid their true value (which is the dominant strategy) in the second-price auction. Thus, the expected profit of the seller (or buyer in the case of procurement auctions) is greater under the first price auction when bidders are risk averse.
Myerson (1981) specified the two types of uncertainty as to why a bidder’s estimates might be unknown to other bidders and to the seller: preference uncertainty and quality uncertainty. The former arises due to inherent differences between preferences of bidders and has been formally documented in literature. However, quality uncertainty has received relatively little attention and Myerson (1981) accounts for the possible impact of other bidders’ value estimates on a bidder’s own estimate by including “revision” functions $e_i(t_i)$ (in addition to his private estimate) in the value function for every bidder.

He characterizes auction mechanisms as a specification of outcome functions $p^*(.)$ and $x^*(.)$, which specify the probability of bidder $i$ winning the auction and the expected payment that he must make to the seller (regardless of the outcome of the auction), along with a description of the strategic plans $\Theta(t_i)$ that bidders are expected to use for the underlying game. Under reasonable assumptions on these functions (including individual rationality and incentive compatibility to make honesty an equilibrium strategy for bidders), an auction mechanism is said to be feasible. An important result illustrated by Myerson is that any feasible auction mechanism is equivalent to some feasible direct mechanism (in which bidders’ expected strategy is to honestly reveal their value estimate) which generates the same expected utilities for all players. This result is formally known as “the revelation principle” and makes the problem of finding optimal mechanisms tractable. Myerson shows that all auctions with the same probability function and expected payoff to the bidder with lowest value estimate will generate the same expected revenue. He defines “priority levels” $c_i(t_i)$ for bidder $i$ which determine his payment as a function of his value estimate under optimality. He characterizes an optimal Vickrey auction as one a second price auction in which the seller submits a bid given by this strategy for his own value estimate.

Harris and Raviv (1981) analyze the problem of choosing the optimal allocation mechanism in different market environments by solving the problem for a simple environment with one seller, two buyers, and a problem to allocate money and bonds between the agents. An important contribution of this paper is in the theorem that any allocation mechanism has an equivalent sealed-bid auction in the sense that they both generate the same equilibrium allocations for the same parameter specifications. They prove that the
optimal mechanism in such a case is similar to the Vickrey competitive auction and that both discriminating and competitive auctions generate the same expected revenue at each possible reservation value.

They also study the optimal allocation design problem when bidders are risk averse and find that the optimal bid in the competitive auction is still equal to the reservation value, but exceeds the equilibrium bid in a discriminating auction due to more aggressive bidding. Moreover, in this case, the expected revenue to the seller is strictly greater in discriminatory auctions than under competitive auctions.

While it is important to study the optimality of auctions across various formats, it is also important to study their optimality in relation to other mechanisms like a negotiated sale or sequential offers. Manelli and Vincent (1995) evaluate the optimality of procurement auctions in maximizing social surplus. They account for the issue of adverse selection associated with these type of auctions by expressing social surplus as a function of the quality of good offered by a seller and the associated probability of trade occurring with that seller. They simplify the linear maximization program subject to Myerson’s incentive compatibility and individual rationality constraints and construct dual variables to solve the problem. This technique of using infinite-dimensional linear programming can be extended to analyze the optimality of other mechanisms as well. An illustrative prediction of their analysis is that sequential offers might be more optimal in environments where the marginal value of quality is of key importance to the buyer to maximize social surplus.

2. Theoretical Argument

Procurement auctions can be modeled as non-cooperative games between multiple selling bidders with one buyer and thus, we have competition on the sellers’ side, instead of the usual case of buyer competition in auctions. Sellers compete on price (and sometimes, quality) of the product in demand and thus, inherent cost advantages are an important factor in determining expected prices in such auctions. Besides the largely idiosyncratic cost component and other private components of a bidder’s valuation of
the good, we assume a common component of the valuation due to characteristics of the good itself. For example, some highway projects might be associated with higher costs for the contractors due to difficult terrain, higher expected traffic or greater exposure to weather patterns like the monsoons. The duration of the project is also directly linked to bidder valuations since longer projects imply greater uncertainty about external risks like regulation and related developments. Thus, longer projects on average are likely to have higher bids across bidders. The competition for that particular job in terms of the number of bidders is also likely to affect the bids submitted by sellers. This effect is dependent on which of the components is deterministic for the auction under consideration – in the case of private values, more competition is expected to encourage more aggressive (lower) bidding, whereas in the case of common values, more competition might raise bids upwards due to the effect of winner’s curse associated with such models. Due to the aforementioned reasons, we model our procurement auctions with an affiliated private values model, and hence, the Milgrom-Weber model of general values will form the basis for further analysis in this paper.

We account for the common component in bidder valuations with a fixed effects regression to identify the collective effect of job-specific factors on bids. The factors include the size of the project, competition, and length of the job. We use fixed effects to account for unobserved heterogeneity across auctions. An implicit assumption here is that bidders are symmetrical in their response to changes in the observable factors, which is potentially restrictive but a reasonable assumption given the relative symmetry of firms in this market because the firms compete in the same sector in the same geography.

The residuals obtained from the fixed effects regression are an estimate for the private component of bidder valuations because they represent idiosyncratic private cost components. We employ the work of Guerre, Perrigne, and Vuong (GPV) to structurally estimate the underlying distribution of valuations without making any parametric assumptions about the functional form. Since they represent private components, the residuals can be used to estimate the underlying game in an independent private values paradigm. Thus, our model is a first-price sealed bid game with independent private values for estimation.
purposes. The GPV estimation procedure has to be modified for our analysis since our data consists of procurement auctions.

Consider the case of a single indivisible project being auctioned to multiple sellers. Let the number of bidders be denoted by \( n \). Each bidder knows his private valuation \( v_i \) of the object, but does not know the valuation of other bidders; he does know that all valuations are independently derived from a common underlying distribution \( F(\cdot) \), which is absolutely continuous on the set of valuations (which we call as support for rest of the paper) \([v_{\text{min}}, v_{\text{max}}]\) where \( v_{\text{min}} \) and \( v_{\text{max}} \) are the lowest and highest valuations of any bidder. Given the distribution of valuations, number of bidders, and his preferences, each bidder submits a bid and the lowest bidder is awarded the project. Since this is a sealed bid auction, bidders are not aware of other bids. It is important to note the following assumptions for the model:

a) Each bidder is identical ex-ante and the game is symmetric

b) Each bidder is risk-neutral

c) Valuations are drawn independently from a common distribution

d) Any variation in the number of bidders is exogenous to the game

An important point to clarify before development of our theory is that the presence of common components in bidder valuations in the general values model does not imply exclusion of the private values of the model. As Hendricks and Porter (2007) point out, we are still in a private values paradigm as long as the distribution of bidder valuations, conditional on a realization of the private component of a bidder’s valuation, is independent of private value realizations for other bidders. In other words, if valuations depend on a known common component, or an unknown component about which bidders have no private information, we are essentially in a private bidding environment since bidders only know about their own signal and are unaware of the private component of other bidders’ valuations. On the other hand, if valuations were determined by an unknown common component about which bidders might have private information, we would be in a common values environment. Thus, the distribution of bidder
valuations, conditional on the private value of a bidder would not be independent of private values of other bidders. Here we assume we are in a private values environment.

Additionally, the general values model does not, in any way, limit the criterion of independence of values in an independent private values (IPV) environment. As illustrated by Milgrom and Weber (1980), statistically independent random variables are always affiliated since the “affiliation inequality” holds with equality for such variables.

Riley and Samuelson’s (1981) characterization of the unique, symmetric, differentiable Bayesian Nash Equilibrium in a static game of incomplete information will guide our theory for the special case of procurement auctions. A bidding strategy $b_i(.)$ is a symmetric equilibrium to a non-cooperative game if it is the best response of bidder $i$, conditional on other bidders $j$ using $b_j(.)=b_i(.)=b(.)$. Thus, we can think of bidder $i$ as choosing his signal $x_i$ and making a bid of $b(x_i)$, and then show that if $b(.)$ is the equilibrium strategy, $x_i=v_i$.

In a procurement auction, bidder $i$’s valuation consists of the costs associated with the job. He will choose a strategy to maximize his expected payoff:

$$E(\pi(b(x_i), v_i)) = (b(x_i) - v_i)(1 - F(x_i))^{n-1}$$

In a procurement auction, the probability of winning is slightly different than in the standard auction. Specifically,

$$Probability(bidder \ i \ wins) = Probability(b(x_i) \leq b) = Probability(x_i \leq x) = (1 - F(x_i))^{n-1}$$

Differentiating our expectation, we get the following differential equation as our first-order condition:

$$1 = \left\{ \frac{f(x_i)}{(1 - F(x_i))} \frac{1}{b'(x_i)} \right\}$$

For $b(v)$ to be the symmetric Bayesian Nash Equilibrium (BNE), $x_i=v_i$. Thus, our equation becomes:
Using this equation, we can express a bidder’s private value as a function of the observed bid, distribution of private values, and number of bidders.

The marginal bidder with \( v_i = v_{\text{min}} \) bids \( v_{\text{min}} \). Thus, our boundary condition is \( b(v_{\text{min}}) = v_{\text{min}} \).

As we can see, the equilibrium BNE for each bidder is strictly increasing in \( v_i \), conditional on the valuation being greater than or equal to the reserve value (which is non-binding for our case).

This expression for the equilibrium bidding strategy of bidders has an associated econometric model because of the relationship between bids and private values. As noted by Laffont and Vuong (1996), for bidder \( i \), his bid \( b_i \) is a function of \( v_i \) (which is a random variable with distribution \( F(.) \)), the bid itself is also a random variable with some distribution \( G(.) \). This observation forms the basis of structural estimation of auction data for both parametric and non-parametric methods. Guerre, Perrigne, and Vuong use the first-order condition for bidder \( i \)’s payoff to express \( F(v), f(v), \) and \( b'(v) \) in terms of \( G(b) \) and \( g(b) \).

Specifically, for every bid \( b \),

\[
G(b_i) = \text{Probability}(b \leq b_i) = \text{Probability}(v \leq v_i) = \text{Probability}(b^{-1}(b_i) \leq b^{-1}(b_i)) = F(b^{-1}(b_i)) = F(v_i)
\]

Thus,

\[1 - F(v) = 1 - G(b)\]

Thus, \( G(.) \) is also continuous with support \([b(v_{\text{min}}), b(v_{\text{max}})]\). The density is given by differentiating \( G(b) \) with respect to \( b \):

\[g(b) = \frac{f(v)}{b'(v)}\]
Thus, we can substitute the following equation into our original differential equation for optimal bidding strategy:

\[
\frac{1}{b'(v)(1 - F(v))} = \frac{g(b)}{(1 - G(b))}
\]

Thus, we have the following characterization of a bidder’s equilibrium bid and his private value in a FPSB auction in an IPV environment:

\[
v = \varphi(b, G, n) = \left( b - \left( \frac{1}{(n - 1)} \frac{1 - G(b)}{g(b)} \right) \right)
\]

Thus, each bidder shades up his bid by a “mark-up factor” to reflect the theoretical prediction that in a first-price sealed bid setting, bidding one’s true value in not the dominant strategy, unlike in a SPSB auction.

We can use the above expression for estimating the latent distribution of bidder valuations, which was the only unobservable parameter of interest for our case. Specifically, the following two step procedure is used for backing out private values:

i. Estimate \( G() \) and \( g() \) by \( G^*(\cdot) \) and \( g^*(\cdot) \) from observed bids non-parametrically. We use a simple step-wise function to estimate \( G^*(\cdot) \) and a kernel density estimator for \( g(\cdot) \). The latter is discussed in more detail in the next section. Given these estimates, we can now compute “pseudo-values” using the reduced-form equation derived above

ii. These pseudo-values form sample realizations of random variable \( V \) with distribution \( F(\cdot) \) and can be used to non-parametrically estimate \( F^*(\cdot) \) and \( f^*(\cdot) \) using the kernel density estimator as in i) above

Even though the function mapping observed bids \( b \) to private value \( v \) is strictly monotonic in \( b \), the identification problem of \( F(v) \) is non-trivial. This is because \( v \) affects the bid in two ways:
- Directly through \( v \), distributed as \( F(.) \)
- Indirectly through the equilibrium bidding strategy \( b(v) \), which depends on the probability of a bidder submitting the lowest bid (since we are considering procurement auctions). This probability in turn, depends on the distribution of private values of other bidders, \( F(.) \)

The identification issue is analyzed by Laffont and Vuong (1996) who prove the uniqueness of \( F^*(.) \) whenever it exists and specify conditions as potential tests for proving the following aspects of the game theoretic model underlying the auction data:

- \( F(.) \) is identified from \( G(.) \) if \( \varphi(b) \) is strictly increasing and differentiable on \([v_*, s(v_2)]\). Porter and Hendricks (2007) also impose boundary conditions on \( \varphi(b) \) in the case of a reserve price set by the seller.
- Moreover, for \( G^*(.) \) to be rationalized by a symmetric IPV game-theoretic model, the bids must be independently and identically distributed as \( G^*(.) \):

\[
G(b_1, ..., b_n) = \prod_{i=1}^{n} G(b_i)
\]

These restrictions imposed by the theoretical model, when applied to data, can be used as a formal test of the theory. Specifically, Guerre, Perrgine, and Vuong (2000) highlight the following equivalence:

- Independence and identical distribution of bidder valuations from \( F(.) \)
- Game-theoretic behavior of bidders by adopting an equilibrium bidding strategy that is strictly increasing in private values

We can use our estimate of the latent distribution of bidder private values to analyze the mechanism design of the auction environment and make recommendations for policy. An important tool that can be used by policymakers to improve auction design is the reserve price to create a floor (in our case, ceiling) for bids. Riley and Samuelson (1981) compute the optimal reservation price to be set by a seller in an IPV
In a procurement auction, let $v_0$ be the buyer’s estimate of the value. In a traditional auction, assuming risk-neutrality of bidders, $v^*$ is the reservation value which maximizes expected revenue of the seller, solves the following equation:

$$v^* = \left\{ v_0 + \frac{1 - F(v^*)}{f(v^*)} \right\}$$

Since we are considering procurement auctions, we adopt a similar methodology and compute the optimal reservation value, above which it is not worthwhile bidding.

Considering the same symmetric IPV model with risk-neutrality, let us analyze the game for bidder 1, without loss of generality:

$$E(\pi_1(v_1, x)) = P(x) - \{v_1 * (1 - F(x))^{n-1}\}$$

Here, bidder 1 choose his signal $x$ and bids $b(x)$. For $b(.)$ to be the equilibrium bidding strategy, it should be optimal for bidder 1 to choose $x=v_1$ and bid $b(v_1)$. $P(x)$ is the expected revenue to bidder 1 in the auction, which may in general include an entry payment even if bidder 1 does not submit the winning bid (although this was not the case in our data). The first-order condition to be satisfied for optimality of bidder 1’s bid is given by:

$$P'(x) - v_1 * \frac{d}{dx} (1 - F^{n-1}(x)) = 0$$

We want to find a reservation value $v^*$ such that a bidder with this valuation is indifferent to entering the auction or staying out. Thus, we get the following boundary condition:

$$E(\pi(v^*, v^*)) = P(v^*) - v^* \cdot (1 - F(v^*))^{n-1} = 0$$

Integrating by parts, we get the following expression for optimal $P(v_1)$:

$$P(v_1) = P(v^*) - \left\{ v^* \cdot (1 - F(v^*))^{n-1} \right\} + \left\{ v_1 \cdot (1 - F(v_1))^{n-1} \right\} + \left\{ \int_{v_1}^{v^*} (1 - F(x))^{n-1} dx \right\}$$
We make use of the boundary condition of $P(v^*)$ to simplify the above expression and we get:

\[ P(v_1) = \left\{ v_1 \left[ (1 - F(v_1))^{n-1} \right] + \int_{v_1}^{v^*} (1 - F(x))^{n-1} \, dx \right\} \]

Let us assume that the cost of doing the job to the buyer is $v_0$. His total expected returns from the auction (including the case in which all bids are higher than $v_0$ and the buyer does the job himself) are given by the following expression:

\[ -\left( v_0 \left[ (1 - F(v_0))^{n-1} + n \left[ \int_{v_1}^{v^*} P(v)f(v)dv \right] \right) \]

Here, $v_*$ is assumed to be the lower bound on bidder valuations.

Substituting for the optimal $P(v)$ and integrating by parts, the seller’s expected return simplifies to:

\[ -\left( v_0 \left[ (1 - F(v_0))^{n-1} + n \left[ \int_{v_1}^{v^*} v(1 - F(v))^{n-1}F'(v)dv + n \int_{v_*}^{v^*} (1 - F(v))^{n-1}F(v)dv \right] \right) \]

The integration limits reflect the fact that in a procurement auction, the reservation value is an upper bound on bidder valuations, as opposed to standard auctions where it is a lower bound on valuations.

Differentiating with respect to $v^*$, we get the following first-order condition:

\[ -\left( v_0 \left[ -(1 - F(v^*))^{n-1}F'(v^*) \right] + n \left[ v^*(1 - F(v^*))^{n-1}F'(v^*) + F(v^*)((1 - F(v^*))^{n-1} \right]\right] = 0 \]

Thus, the optimal reservation value above which it is not profitable to bid in a procurement auction is given by the following expression:

\[ v^* = \left( v_0 - \frac{F(v^*)}{f(v^*)} \right) \]

A few implications of this result include:
• The optimal reservation value for a procurement auction in the IPV paradigm is independent of the number of bidders or the equilibrium bidding strategy adopted by bidders

• It is applicable to FPSB auctions, and hence to the auctions in our sample

• Since we estimate distribution of bidder valuations non-parametrically, our estimates for optimal reserve value are robust to misspecification issues associated with parametric estimation

3. Data

The Municipal Corporation of Greater Mumbai (“MCGM”) invites online tenders for public works on a regular basis from both firms registered with the corporation and other government bodies engaged in infrastructure development. Our data only deals with procurement auctions for improvement and construction of various roads in the city of Mumbai. All interested vendors are provided with a list of works to be auctioned along with the estimated cost, duration, and associated tender fees. Detailed requirements for actual implementation of the project (including type of raw materials used, variations allowed in the quality of materials used, and number of managerial officers employed on site) are also provided in the tender document. Each work is given a unique identification number used to distinguish otherwise similar works in different regional wards, and each submitted bid is also assigned a unique bid number.

Contractors have to submit documentation in three packets – Packets A, B, and C. Packet A contains documents to verify identity and registration of the tenderer including Bank Solvency Certificate and Sales Tax Registration Certificate. Packet B contains documentation to assess the current operations of the bidder including evidence of recent civil works undertaken and their outcomes and audited balance sheets for last three years. Packet C comprises the financial bidding information including the online tender with the quoted rate, required amount for Additional Security Deposit for rebates exceeding 12%, and a rates analysis if the rebate exceeds 12% or premium exceeds 15%. Only bids from contractors who
satisfy the requirements in Packets A and B, as declared by a designated MCGM official, are opened in Packet C. Moreover, bids which are missing the rates analysis are automatically disqualified from the process. Our data indicates that bids at a rebate greater than 12% or premium greater than 15% were declared non-responsive in the case of multiple tenders.

Specifically, a bid is declared “responsive” if it conforms to all terms, conditions, and specifications contained in the tender document, without “material deviation” or reservation. Material deviation or reservations are those that:

- Affects the scope, quality, or performance of the work being auctioned
- Limits the Employer’s rights or the tenderer’s obligations under the contract (if awarded)
- Would affect fairness of the auction and the competitive position of other bidders

Bids are submitted by firms on a “rate basis” i.e. as a discount or premium to the estimated cost of the work provided by the corporation itself. Although rebates greater than 12% of this cost and premiums greater than 15% are subject to special arrangements as mentioned above, we do not take these thresholds to be the reservation price for the single-unit auctions. Rebates greater than 12% of the office cost estimates are subjected to an “Additional Security Deposit” (3% of estimated cost). This deposit is subsequently returned to the tenderers after evaluation of tenders, and hence, can be considered as a mechanism for ensuring creditworthiness of the contractor.

On the opening date, authorized officials access the online portal system that generates a ranking of responsive bids conforming to the document according to the bids submitted. The lowest responsive bid in conformity with the tender document is awarded the contract and basic execution is expected to begin within 15 days. In comparing tenders, efficiency and reliability of the bidder are considered in addition to the eligibility criteria mentioned in preceding paragraphs. The winner is required to provide deposits with MCGM including Contract Deposit (2% of the contract amount) and Retention Money (5% of the bill
amount), in addition to fees for various administrative tasks; these liabilities are included in total cost of the project to the contractor for the purposes of this paper.

We have data on a total of 67 procurement auctions for improvement of city roads in Greater Mumbai during the years 2006-2007, 2008-2009 and 2009-2010. The data includes information on the actual observed bids, length of project, estimated costs, and number of bidders. Four of these auctions only had one observed bid and hence were dropped from the sample. Details of responsive and non-responsive bids for the three sub-samples of our data are provided in Table 1 below:
Our sample is essentially in the form of panel data where \( \{X_{i,t}\} \) represents the realization of variable X for bid \( i \) in auction \( t \).

**Data Modeling**

The procurement auctions in our data can be modeled as first-price sealed bid static game of incomplete information in the Milgrom-Weber affiliated values paradigm. We also assume risk-neutrality of tenderers with respect to winning the contract thus, are able to employ the results for models assuming risk-neutrality of bidders as those contained in Laffont and Vuong (1996), Guerre, Perrigne, and Vuong (2000), and Riley and Samuelson (1981). There is no publicly declared reserve price for the auctions, although there is a nominal entry fee (“cost of E-Tender”) for each auction. This fee, however, constitutes an insignificant percentage of the estimated cost of projects and observed bids and is not expected to have
a significant impact on our analysis. Since we assume bidders to have non-negative valuations, the implicit reserve price of zero is non-binding; this ensures that the number of potential bidders is equal to the number of actual bidders (ignoring costs imposed on bidders in getting information about the job and the market).

In procurement auctions, bidder valuations represent the costs incurred as a result of undertaking the project, instead of the value accorded to the good, as is the case in standard auctions. These costs include a common component dependent on specific job-characteristics, affecting utilities of all contractors, and an idiosyncratic private component dependent on firm-specific factors like technological capability, location, and size of the contractor. Thus, we can use the Milgrom-Weber affiliated values model for further analysis in this paper. We assume that these valuations are distributed independently across firms – firms know their own costs, but only know the distribution of costs of other firms. This assumption is similar to that of Porter and Zona in their analysis of highway procurement auctions in Long Island (1992).

Additionally, we assume the number of potential bidders for each auction to be common knowledge for the purposes of this paper, but the possible endogeneity of the number of bidders is an area for further research with our dataset. Bidders are assumed to be symmetric and the distribution of bidder valuations is symmetric in its components. Another potentially restrictive assumption in our model is that the private cost component is one-dimensional since this eliminates the possibility of bidders having varying information on different components of their payoff. (Porter and Hendricks, 2080).

4. **Empirical Model**

We use multi-variable regressions to study the effect of auction-specific observable factors on observed bids. Specifically, we use a fixed-effects (“FE”) model to capture the effect of all auction-specific information on bids and obtain residuals which are our estimates for private components to be used in the GPV expression. Next, we use predicted value from the FE model as the dependent variable for a
generalized-least squares regression ("GLS") to estimate the effect of specific observable factors including the number of bidders, estimated cost and time duration of the job. We use GLS since the estimates from the FE model are equal to the sample average and as a result, the error terms from ordinary-least squares ("OLS") are likely to be heteroskedastic.

Thus, our regressions are as follows:

- For FE, we include a dummy variable for each auction and obtain predicted values for $Lb_{i,t}$ the log of the bid of bidder $i$ in auction $t$

  $Lb_{i,t} = \beta_t + \epsilon_{i,t}$

- We next use these predicted values as the dependent variables in the following GLS to estimate the auction-specific component of bidder valuations, $p_{-bid_t}$, where $p_{-bid_t}$ is the winning bid:

  $$p_{-lb_t}/N_t^{1/2} = \beta_0 + \beta_1 Lcost_t/N_t^{1/2} + \beta_2 N_t^{1/2} + \beta_3 L_t/N_t^{1/2} + \epsilon_t/N_t^{1/2}$$

A brief description of the regression variables, along with their expected signs are provided in Table 2 below:
The expected sign for $L_{cost}$ is positive since the estimated size of the project should roughly correspond to the value to private bidders of completing the project. Thus, we expect $\beta_1$ to be strongly positive.

The relationship between the winning bid and the number of bidders differs under different information spaces. If the independent private component of bidder valuations is the deterministic factor for the auction, a greater number of bidders implies greater competition, and hence, lower bids for the job. However, if the common component is the deterministic factor, more competition might not necessarily imply more aggressive bidding. The effect of an increase in competition is counterbalanced by the effect of winner’s curse in a common values auction. The magnitude of $\beta_2$ can also be used to implement policy that will increase or decrease competition in the market, depending on the particular case.

Lastly, the estimated duration of the project is likely to have a positive correlation with the submitted bids because bidders face a bigger opportunity cost for their resources with longer projects, even after controlling for the size of the project (through $L_{cost}$). Moreover, the costs incurred due to delays in the monsoon season are also greater for longer projects.
The purpose of the regression in context of this paper is two-fold:

- Estimate the common component of bidder valuations and compute estimates for the private value by using the residuals of the regression. The underlying game is now in the independent private values paradigm and we can employ relatively simplified results for these games to our data
- Study the relationship between bids and observable job-specific factors like number of bidders and project duration to propose recommendations for policy objectives like maximizing expected revenue

Our next step is to estimate the underlying distribution of bidder costs using the two-step method proposed by Guerre, Perrigne, and Vuong:

- We use predicted values for $Lb_{i,t}$, $p_{Lb_{i,t}}$ from the FE model to compute estimates for the private component of the observed bids to be used in the GPV expression:
  \[ p_{res_{i,t}} = b_{i,t} - e^{p_{Lb_{i,t}}} \]
- The non-parametric estimation of the distribution of pseudo-private values and observed bids is done using a kernel density estimator:
  \[ g^*(b) = \frac{1}{NT} \sum_{i=1}^{T} \sum_{p=1}^{N} K_g \frac{b - B_{p,t}}{h_g} \]
  
  For our computation in STATA, $K_g$ is the Epanechnikov kernel function and $h_g$ is the optimal bandwidth that would minimize the mean integrated squared error for Gaussian data.
- $G^*(b)$ is computed from the empirical distribution using a simple stepwise function.
- The next step is to back-out the private component of bidder $i$’s valuation in auction $t$ from the bids by using the GPV expression:
are our estimates for “pseudo-private values” of bidders.

- We now want to include the auction-specific element into our estimate of bidder $i$’s valuation:

$$v_{i,t} = v'_{i,t} + p_{bid_t}$$

Here, $p_{bid_t}$ is the predicted value for auction $t$ obtained from our GLS regression.

- The final step is to estimate the underlying distribution of $v_{i,t}$, $f^*(v)$, using a kernel density estimator similar to the one used to estimate $g^*(b)$.

$F^*(v)$ is then calculated from $f^*(v)$.

- Once we have our estimates for the latent distribution of bidder valuations, it is possible to compute the optimal reservation value for the auctions using the expression derived in our theory:

$$v^* = \left( v_{0,t} - \frac{F(v^*)}{F'(v^*)} \right)$$

Here, $v_{0,t}$ is the value of the project to the buyer. Since our data does not directly include this value, we use the estimated cost of job $t$ provided by MCGM as a proxy and estimate $v^*$ for a range of values $v_0$ around the estimated cost for each auction. We are thus able to model the optimal reserve value for our data, given the common underlying distribution of values, $F(.)$, which is an important prediction for the policy objective of maximizing expected government revenue.
5. **Regression Results:**

**FE model:**

Estimates for the coefficients on auction dummies from the FE regression are not included here since we are only interested in the residuals from this regression for further analysis.

**GLS Model:**

As expected, an increase in the number of bidders (while controlling for the size of projects through inclusion of estimated cost as an explanatory variable in the regression) is correlated with an approximately 5.1% decline in bids because of aggressive bidding in a more competitive environment. This result is also in agreement with the general effect of competition in a private values environment.

The large positive coefficient on $L_{cost}$ is indicative of the direct relationship between estimated cost of a job and submitted bids.

The sign on $L_i$ is positive as expected, but project length is not statistically significant in explaining the common component of observed bids. This might be the result of limited variation in project durations for the auctions in our dataset, or because some of the additional costs associated with longer projects are already included in the estimated cost.
Non-parametric Estimation of Density Functions:

The kernel density estimator discussed in prior sections gives us the following estimates for $g(b)$ and $f(v)$.

Distribution of observed bids:
Distribution of bidder values:

Kernel Density Estimate for Bidder Valuations

Estimates of Bidder Values

kernel = epanechnikov, bandwidth = 1.1e+07
To compute the optimal reserve values for auctions in our sample, we perform a sensitivity analysis using our estimates of $F(.)$ and $f(.)$. Our results thus comprise of $v^*$ for a range of $v_0$, for each auction. The optimal reserve value, as a percentage of the buyer’s reserve value, is centered on 58-72% across auctions, as shown in the chart below:
We now have a set of values of \( v^* \) corresponding to \( v_0 \), determined by our implicit formula for \( v^* \).

To approximate an explicit function for \( v^* \) as a function of \( v_0 \), we fit values of \( v^* \) as a function of \( v_0 \) for linear and quadratic models respectively:

\[
v^* = \alpha + \beta_1 v_0 + \epsilon
\]

\[
v^* = \alpha + \beta_1 v_0 + \beta_2 v_0^2 + \epsilon
\]

The quadratic model fits our estimates with greater accuracy as evident from the regression results. Specifically, the R-2 associated with the second-order model is 0.96 as compared to an R-2 of 0.88 for the first-order model, as evident from the regression results produced in the next page.
Quadratic model:

```
reg v v0 v0_2

Source | SS     | df  | MS        | Number of obs = 315
Model   | 4.9387e+17 | 2   | 2.4694e+17 | F(2, 312) = 3830.17
Residual| 2.0115e+16 | 312 | 6.4472e+13 | R-squared = 0.9609
Total   | 5.1399e+17 | 314 | 1.6369e+15 | Adj R-squared = 0.9606
```

```
|       | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|-------|--------|-----------|-------|-----|---------------------|
| v0    | 1.012097 | 0.0216997 | 46.64 | 0.000 | 0.9694006         | 1.054793 |
| v0_2  | -1.30e-09 | 5.29e-11  | -24.55| 0.000 | -1.40e-09         | -1.19e-09 |
| _cons | -1.75e+07 | 2060304   | -8.47 | 0.000 | -2.15e+07         | -1.34e+07 |
```

First-Order Model:

```
.reg v v0

Source | SS     | df  | MS        | Number of obs = 315
Model   | 4.5501e+17 | 1   | 4.5501e+17 | F(1, 313) = 2414.89
Residual| 5.0976e+16 | 313 | 1.6842e+14 | R-squared = 0.8853
Total   | 5.1399e+17 | 314 | 1.6369e+15 | Root MSE = 1.4e+07
```

```
|       | Coef.  | Std. Err. | t     | P>|t|  | [95% Conf. Interval] |
|-------|--------|-----------|-------|-----|---------------------|
| v0    | .4997585 | .0101698  | 49.14 | 0.000 | 0.4797487         | 0.5197683 |
| _cons | 2.49e+07 | 1928387   | 12.90 | 0.000 | 2.11e+07         | 2.87e+07  |
```
A plot of the actual values versus the fitted values further illustrates this point:
6. **Conclusion**

Since we are in a private values environment, the Revenue Equivalence Theorem holds and the expected revenue, conditional on the reserve value, is the same from all four auction formats. Thus, setting a minimum reserve value as predicted by our model will maximize expected revenue for the government for such procurement auctions.

Since we have estimated the underlying distribution of bidder valuations using non-parametric methods, and hence, our results are relatively robust and independent of its specific functional form. Moreover, we have accounted for auction-specific common factors affecting bidder valuations through our fixed-effects model even in a private values paradigm. This unique approach of combining an econometric model with theoretical predictions for the underlying game provides us with relatively solid results which are of importance from a policy perspective.

The government can also look into subsidizing entry for smaller firms to increase competition and encourage aggressive bidding. This can be done by providing a subsidy to the smaller firms or including limited financial assistance to the winner in the tender document. This recommendation, however, requires further research on the entry decision of firms since variation in number of bidders is not exogenous in some auction settings. McAfee and McMillan (1986) analyze auctions where bidders can enter on paying an entry fee and find that the optimal auction design with endogenous entry is a FPSB auction in which the reservation value is equal to the seller’s own value. Since we are looking at procurement auctions, it is also important to consider the impact of any such subsidy on the potential issues of adverse selection and moral hazard.

Given our recommendations for policy, it is also important to note the limitations of our analysis. We have assumed risk-neutrality and symmetry of bidders for our theoretical framework and validity of these assumptions are crucial to our results and relaxing these behavioral assumptions is likely to significantly affect results. Moreover, some procurement auctions including auctions for mineral rights and natural gas
and oil tracts cannot be modeled under a private values paradigm since the common component of bidder valuations is not independent of specific realizations of bidder valuation. Thus, estimating optimal reserve value for common values auctions in the region is another area for further research.

**References:**
