The Robustness of Herrera, Levine and Martinelli’s Policy platforms, campaign spending and voter participation*

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Abstract

In this paper, I look at potential weaknesses in the electoral competition model constructed by Herrera, Levine, and Martinelli in their paper *Policy Platforms, Campaign Spending and Voter Participation* (Herrera, Levine, and Martinelli, 2008). In the original paper, the authors seek to explain why campaign spending has increased at a time when US politics has become more polarized. To do so, Herrera et al. develop a game theoretic model of electoral competition where parties compete in an election by choosing their policy platform and an amount of campaign spending. Using the results from their model as evidence, the authors hypothesize that the increase in campaign spending and polarization in United State politics can be attributed to an increase in the volatility of individual voters’ preferences. This finding goes against the popular explanation that campaign spending has been increasing because of improvements in voter targeting techniques. After relaxing some of the assumptions in the original model, I was unable to disprove the authors’ findings. However after thoroughly examining the model, I found inconsistencies between the model and real world behavior that led me to question the accuracy of the authors’ results.

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1 Introduction

Citing findings by McCarty, Poole, and Rosenthal (2006), Corrado, Mann, Ortiz, and Potter (2005), and party identification data from The American National Election Studies (2007), Herrera, Levine, and Martinelli found the following three trends in American politics since the middle of the 20th century:

1. Increased polarization between Democratic and Republican legislators,
2. Increased campaign spending, and
3. Decreased party loyalty among individual voters.

In an attempt to explain these trends, Herrera et al. develop a game theoretic model of electoral competition in which parties compete for voters in an election by choosing their policy platform and an amount of campaign spending. Using their model as evidence, the authors refute the common view that campaign spending has increased as a result of improved targeting techniques. Instead, they hypothesize that the rise in campaign spending and the increase in polarization in US politics can be attributed to an increase in the volatility of voter preferences, which they define as “large aggregate shocks to party bias.” In Herrera’s model, a rise in voter targeting leads only to an increase in campaign spending, while a rise in voter volatility leads to both a rise in campaign spending and a rise in political polarization. According to the authors, an increase in volatility makes the results of an election less predictable. As a result, parties have less of an incentive to move to the median voter and end up moving closer to their own ideal point. Because parties begin to move towards opposite sides of the political spectrum, each party has more to lose by losing the election. Thus, winning the election becomes more important, and parties spend more as a result.

Though the model does support the authors’ hypothesis, Herrera et al. assume in the model that both parties are equally effective at spending money to target voters, an assumption that is unrealistic. In the 2004 election, George Bush spent only $60 million more than John Kerry, but Bush was able to spend more effectively, mobilizing more of his core supporters to vote en route to his 2004 victory (Edsall and Grimaldi, 2004). Unsatisfied with Herrera’s assumption concerning both parties effectiveness at targeting voters, I reexamined Herrera’s model allowing each party to be independently effective at utilizing its resources.

Like much of the current literature, Herrera’s model builds off of the work of Wittman (1983) and Calvert (1985) and assumes that candidates derive utility from both winning office as well as the policy that is enacted. The assumption that candidates are policy
driven allows for candidates to choose distinct policy platforms, a departure from the median voter theorem popularized by Downs (1957). Herrera’s model distinguishes itself from the current literature by focusing on campaign spending as a means to mobilize voters instead of to influence them. Most of the current literature modeling electoral competition views campaign spending as a means to influence voters’ perceptions or preference for a given candidate. For example, Coate (2004) and Schultz (2007) view campaign spending as a way to lessen informational imperfections by informing voters of each candidate’s policy platform. In Coate’s model, Coate assumes imperfect information between candidates and voters so that voters do not know each candidate’s exact position. In the model, parties first choose a candidate. Interest groups for both parties see the candidate choice and choose how much they want to contribute to the campaign. The candidate then uses these campaign contributions in order to inform voters of his platform, lessening the effects of imperfect information. The winning candidates then enacts his preferred policy. In Schultz’s model, voters again do not know each candidate’s exact position. In the model, candidates simultaneously choose their position and set an advertising budget. Advertisements reach a given sector of voters. Voters then update beliefs and vote for the candidate they perceive as best. In both models, voters are not aware of the candidates’ positions, and campaign spending is used to lessen the informational asymmetry.

Meirowitz (2008) and Ashworth and Bueno de Mesquita (2007) also examine the effects of campaign spending, but in their models, campaign spending is used to invest in valence. Meirowitz models the election between a challenger and an incumbent. At the start of the game, nature gives the incumbent an advantage, and then both the incumbent and the challenger choose a given level of valence. Voters vote for their preferred candidate taking into account each candidate’s investment in valence and the inherent advantage that the incumbent is given. In Ashworth and Bueno de Mesquita’s model, candidates choose both a policy platform and a level of investment in valence. In the model, the importance of valence and the distance between the two candidates are inversely proportional, so as the difference between policies shrinks, the importance of valence increases. Both models view campaign spending as an investment in valence, which increases candidates’ appeal to potential voters.

In Herrera’s model, campaign spending has no influence on voters’ perceptions of a given candidate. Instead, campaign spending affects the percentage of a candidate’s supporters who actually go out and vote. In the model, the election takes place in two stages. Each party chooses its platform policy in the first stage and chooses how much to spend on the campaign in the second. Voters decide between the two parties based
on their own individual ideology as well as an individual and a societal bias towards one party or the other. Voter turnout for each party is then determined as a function of the number of voters supporting each party and the campaign spending of each party.

In the model, the writers includes a variable $t$ meant to represent both parties’ effectiveness at spending money to target voters. They assume $t$ to be the same for both the Democratic and Republican parties. I reexamine the model by relaxing the symmetry in $t$ and allowing for party specific $t$ values. By doing so, I make the model more realistic, which in turn gives more strength to its findings.

After allowing for party specific $t$ values, I found that the model still supports Herrera, Levine, and Martinelli’s theory that the increase in campaign spending and polarization in American politics can be attributed to increased volatility among individual voters. However, in equilibrium, the model predicts behavior that is inconsistent with what occurs in real world elections. I prove that, even in the general case, both parties find it optimal to choose symmetric policy platforms and equivalent levels of campaign spending. This behavior occurs even if one party is given inherent advantages over the other. The symmetry between the actions of the two parties, especially when one party is given an advantage over the other, is behavior that does not happen in real world elections and makes me question the results derived from the model.

2 The Model

2.1 Herrera, Levine, and Martinelli’s original model

In Policy Platforms, Campaign Spending, and Voter Participation, Herrera et al. model elections as a winner takes all two stage simultaneous move game between players $D$ and $R$. In the first stage of the game, $D$ and $R$ choose binding policy platforms $d$ and $1 - r$ respectively where $d$, $1 - r$ are elements of a given policy space $[0,1]$. $D$ and $R$ next choose a level of campaign spending $D$ and $R$ respectively where $D$, $R$ are elements of an effort space $[0,1]$.

The model assumes that $D$ and $R$ gain utility both from the policy that is enacted as well as an inherent utility, $G$, from winning the election. $D$ and $R$ are defined to have Euclidean preferences and ideal points at 0 and 1 respectively on the policy space. The utility of each party is also negatively affected by their campaign spending. The disutility of campaign spending is equal to the amount of spending itself. Thus, if a party spends $C \in [0,1]$, the party’s utility is affected $-C$. The payout function for each party is as follows:
Herrera’s model builds off of a previous model developed by Lindbeck and Weibull (1993) where the election is decided by a continuum of voter that have both a policy preference as well as an individual party bias. Voters are indexed by their ideal point $v$, where $v \in U[0, 1]$. All voters have a societal aggregate party bias, $b$ uniform over $[-\beta, \beta]$, in favor of $D$ as well as an individual party bias, $b_v$ uniform over $[-\alpha, \alpha]$, in favor of $D$. Taking all of these factors into account, a voter $v$ favors party $D$ if

$$|v - d| + b_v + b > |v - (1 - r)|.$$ 

Voter $v$ will favor party $R$ when the inequality is reversed. The voter would be indifferent between the two parties in equality, but because $b$ and $b_v$ are drawn from continuous distributions, equality can be ignored. Also, it is assumed $\alpha > 1$, so regardless of a voter’s ideal policy, it is possible for the voter to support either party.

After the preference of each individual voter has been realized in the first period, the winner of the election is determined by the number of voters supporting each party and the amount of campaign spending done by each party. The model assumes that not all voters actually go out and vote for their preferred candidate. Instead, the number of voters for a party is dependent on the amount each party spends to mobilize its voting block. In the original model, Herrera defines the variable $t \in [1/2, 1]$ to be the accuracy of campaign targeting and calculates that a fraction $tD + (1 - t)R$ supporters of $D$ come out to vote while a fraction $(1 - t)D + tR$ supporters of $R$ come out to vote. The rest are assumed to abstain. The model assumes that if party $D$ spends $D$ on mobilizing voters, $D$ successfully reaches $tD$ percent of his base, while $(1 - t)$ backfires, mobilizing $(1 - t)D$ percent of $R$’s base.

### 2.2 Adding party specific $t$ values to the original model

In the original model, Herrera et al. assume both parties are equally effective at targeting their voting base. To test the robustness of the model, I relaxed this assumption and allowed for party specific values of $t$. I define $t_D, t_R \in [1/2, 1]$ as the effectiveness of $D$
and \( R \) at mobilizing its base, so assuming that a number \( S_D \) supports party \( D \) and \( S_R \) supports party \( R \), the number of voters for \( D \) and \( R \) are \((t_D D + (1 - t_R)R) \times S_D\) and \(((1 - t_D)D + t_R R) \times S_R\). Thus, in the second stage of the game \( D \) wins if

\[
(t_D D + (1 - t_R)R) \times S_D > ((1 - t_D)D + t_R R) \times S_R
\]

and \( R \) wins if the equality is reversed.

There are two noteworthy aspects concerning the determination of the winner of the election. First, as noted in the original paper, campaign spending affects the fraction of voters mobilized, not the absolute number of voters mobilized. Thus, a party receives a disutility of \(-1/2\) to mobilize half of its supporters regardless of the size of its voting base. The authors view voter mobilization as providing information about voting and stressing the importance of voting through advertising in order to motivate potential voters to vote.

Secondly, in my adjusted version of the model, a party could potentially mobilize over 100\% of its voting base. For example, if \( t_D = 1, \ D = 1, \ R \neq 0, \) then \( t_D D + (1 - t_R)R > 1, \) and \( D \) would receive more votes than the size of its voting base. In order to rationalize this case, I assume that there are two groups of voters. One is the group of voters discussed above that are motivated by policy and party affiliation. The second is a group of passive constituents that usually do not vote but can be swayed to vote for one party or the other by aggressive advertising. Thus, in the example above where \( t_D D + (1 - t_R)R > 1, \) all of the policy driven voters supporting party \( D \) turn out to vote for \( D \) as well as an additional group of voters who come out to vote for \( D \) strictly because of a combination of the campaign spending of \( D \) and the backfire from the campaign spending of \( R \).

3 Equilibrium using party specific \( t \) values

In the following model, I have incorporated the party specific \( t \) values discussed above and have used a framework identical to the one used by Herrera et al. to solve model.\(^1\) All proofs have been included in the Appendix.

**Theorem 1.** The fraction of voters favoring party \( D \) is

\[
1/2 + (b + d - d^2 - r + r^2)/(2\beta)
\]

\(^1\)At any point in the adjusted model, the expression \( t_R = t_D = t \) can be substituted to ensure that this model is consistent with the findings of Herrera’s previous model.
with the remainder
\[ 1/2 - (b + d - d^2 - r + r^2)/(2\beta) \]
favoring party R. If \( D + R > 0 \), the probability that party D wins is
\[ F(d - d^2 - r + r^2 + \frac{\beta((2t_D - 1)D - (2t_R - 1)R)}{D + R}) \] (1)
where \( F \) is the distribution function of the variable \( b \).

Aggregate voter turnout is
\[ \frac{D + R}{2} + [(2t_D - 1)D - (2t_R - 1)R](b + d - d^2 - r + r^2)/2\beta \]
and the winning margin is
\[ \frac{(2t_D - 1)D - (2t_R - 1)R}{2} + (D + R)(b + d - d^2 - r + r^2)/2\beta. \]

Because the fraction of voters favoring party D is unaffected by \( t \), the first part of Theorem 1 is identical to that of the original model. The only difference between the first part of this model and the first part of the original model is the term \((2t_D - 1)D - (2t_R - 1)R\) which replaces the term \((2t - 1)(D - R)\) from the original model.

Using the probability of winning derived from Theorem 1, the equilibrium solution to campaign spending in stage two can be found given policy platforms.

**Theorem 2.** If \( 1 - d - r + G \leq 0 \), then the unique second stage Nash choice of campaign spending is \( D = R = 0 \). Otherwise, both parties spend
\[ D^* = R^* = \min\{\beta(\frac{t_D + t_R - 1}{2})(1 - d - r + G)/2\alpha, 1\}. \]

Similar to the original model, both parties find it optimal to spend the same amount given any choice of policy platforms and any realization of \( t_R, t_D \). The symmetry behind this result is discussed in section 4.

Given the results found in Theorem 1 and the optimal amount of spending in the second period, it is possible to calculate the optimal policy platform of each party in stage one. To stay consistent with the original model and avoid dealing with corner solutions, assume that:
\[ \beta(\frac{t_D + t_R - 1}{2}) < \alpha < 1 + G + \beta(\frac{t_D + t_R - 1}{2}) \] (2)

As will be seen by the equilibrium solution, this inequality provides enough electoral uncertainty that both parties refrain from converging to the median voter, but ensures
that there is not so much uncertainty that both parties simply choose their preferred policy. Combining Theorem 1, Theorem 2, and Inequality 2 leads to the following:

**Theorem 3.** If Inequality 2 holds, there is a unique subgame perfect Nash equilibrium, it is symmetric, and in equilibrium each party chooses the platform

\[
d^* = r^* = \frac{1}{2} - \frac{1}{4} \sqrt{G^2 + 4(\alpha - \beta (\frac{tD + tR - 1}{2}))} - G
\]

where \(0 < p^* < 1/2\).

4 Comparison of results between adjusted model and original model

Except for the substitutions of \((2t_D - 1)D - (2t_R - 1)R\) and \(\frac{t_D + t_R}{2}\) in parts of the new model, equilibrium in the updated model mirrors equilibrium in the original model. Both parties make symmetry decisions for campaign spending in stage one and policy in stage two and increased campaign spending and polarization can be attributed to increased volatility in voter preferences, as hypothesized in the original paper.

The symmetric choices for campaign spending and policy in this updated version of the model was suprising because it means that in equilibrium, the party that is at a disadvantage in targeting voters does nothing to make up for its disadvantage. Looking at equation (1) illustrates this point. When substituting \(D = R\) and \(d = r\) into equation (1), the probability that party \(D\) wins becomes

\[
F(t_D - t_R).
\]

Intuitively, if one party has an advantage in targeting partisan voters, the other party would be expected to make up for its disadvantage either by moving its policy towards the median voter in stage one or spending more in stage two, but in this model, no such adjustment is made. Both parties make symmetry policy and campaign spending decisions, and ultimately the party with the disadvantage is content with having a lower probability of winning.

After discovering this symmetry even in the adjusted model, I examined both parties’ choices for campaign spending and policy in the general case. I was able to prove for the general case, included in section 2 of the appendix, that in stage two both parties find it optimal to spend the same amount. Using the equivalence between \(D\) and \(R\) in stage two, it is then easy to prove that given \(D = R = x(1 - r - d + G)\) where \(x\) is
some constant, the unique subgame perfect Nash equilibrium in stage one is \( d^* = r^* \).

To explain the importance of \( D = R = x(1 - r - d + G) \), if \( d \) or \( r \) increases, either the party’s proposed policy is moving away from the party’s ideal point or the opponent’s proposed policy is moving towards the party’s ideal point meaning that parties have less of an incentive to spend. If \( G \) increases, then the payoff for winning increases, and both parties have a greater incentive to spend. By examining each party’s choice for campaign spending and policy in the general case, it is clear that the symmetric results found in the party specific \( t \) variation of the model are not a coincidence but instead illustrate a greater symmetry within the model. Both parties will make symmetry decisions for campaign spending and policy, and the disadvantaged party appears content with its lower probability of winning.

With campaign contributions for presidential elections rising every election cycle since 1980 and 2008’s level of campaign contribution nearly doubling 2004’s record setting level (Center for Responsive Politics, 2009), political elections continue to grow in scope. The defeatist attitude of the disadvantaged party predicted by the model is a far cry from the aggressive nature of real political elections. Furthermore, the spending tendencies of candidates predicted by the model are fundamentally inconsistent with the spending tendencies of candidates in an election. According to the Federal Election Commission, in the 2008 election John McCain raised $384 million and spent $358 million while Barack Obama raised $778 million and spent $760 million. In real world elections, candidates look to raise and spend as much money as possible. Candidates do not limit themselves the way that the model predicts. After closely examining the model, the model’s strict symmetry and unrealistic proposed behavior produces serious doubts as to whether the model is an accurate depiction of electoral competition and seriously weakens Herrera, Levine, and Martinelli’s hypothesis on current political trends.

5 Conclusion

In the original model developed by Herrera, Levine, and Martinelli, the authors attempt to explain the rise in political spending and the simultaneous increase in polarization in the US. Their original model counters the common explanation that these trends are caused by an increased ability to target potential voters by showing that an increase in targeting ability increases spending but decreases polarization by moving candidates

\[ \text{This can be done by substituting } D = R = x(1 - r - d + G) \text{ into the proofs of Lemma 1-4 and Theorem 3 in part 1 of the Appendix.} \]

\[ \text{The Center for Responsive Politics is a research group founded in 1983 by senators to track money in US politics.} \]
closer to the median voter. The authors conclude that the increase in spending and polarization is caused by the third emerging trend in US politics; the increase in the volatility of voter preferences. In Herrera’s model, increasing voter volatility leads parties to spend more in order to ensure victory and also leads candidates to move closer to their ideal points on the two extremes of the political spectrum.

After examining the model, I attempted to relax the equality on each party’s ability to target its voting base and see if the model’s results still held up. While the model does come to the same conclusion concerning the explanation of current political trends, relaxing this equality constraint also shows behavior that appears to be inconsistent with real world elections. I was able to prove an inherent symmetry within the model which led both parties to make symmetry campaign spending and policy decisions in equilibrium, even when one party was given the disadvantage of a worse ability to target its base of potential voters. The model showed the disadvantaged party essentially conceding defeat in this area and accepting its lowered probability of winning, a far cry from the win at all costs feel that characterize most competitive political elections.

Though Herrera, Levine, and Martinelli develop an innovative explanation of current political trends, the strict equality between $d$ and $r$ and $D$ and $R$ displays a lack of competition between the two parties inconsistent with current political elections. Because in a typical election a candidate will spend the majority of the resources at his disposal, further work can be done to improve the costs of campaign spending and the payout structure in an attempt to break the symmetry between $D$ and $R$. Herrera, Levine, and Martinelli’s paper presents a solid first step in modeling electoral competition and looking at the current trends of American politics, but the model’s inability to accurately simulate the behavior and asymmetries found in political elections make it unable to solidly back up their claim.
Part 1 of the appendix includes the proofs of Theorems 1-3 necessary to solve the electoral competition model. Part 2 of the appendix includes the proof that \( D = R \) discussed in the comparison between the updated model and Herrera et al’s original model.

### A Solution of Adjusted Model

#### A.1 Proof of Theorem 1.

Using Eq. (1), if \( d \leq 1 - r \),

\[
\Pr\{\text{voter } v \text{ favors party } D\} = \begin{cases} 
\frac{1}{2} + \frac{1-r-d+b}{2\beta} & \text{if } 0 \leq v \leq d \\
\frac{1}{2} + \frac{1-r+d+b-2v}{2\beta} & \text{if } d \leq v \leq 1 - r \\
\frac{1}{2} + \frac{r-1+d+b}{2\beta} & \text{if } 1 - r \leq v \leq 1
\end{cases}
\]

By integrating the above over all voters \( v \in [0, 1] \), the total fraction favoring \( D \) is

\[
d(\frac{1}{2} + \frac{1-r-d+b}{2\beta}) + (1-r-d)(\frac{1}{2} + \frac{1-r+d+b-2v}{2\beta}) + r(\frac{1}{2} + \frac{r-1+d+b}{2\beta}) + \int_d^{1-r} \frac{-2v}{2\beta} dv \\
= \frac{1}{2} + \frac{b+d-d^2-r+r^2}{2\beta}.
\]

Similar calculations show the result is the same when \( d > 1 - r \). The fraction of voters supporting \( R \) is \( \frac{1}{2} - \frac{b+d-d^2-r+r^2}{2\beta} \), so the probability that \( D \) wins is equal to the probability that

\[
(t_D D + (1-t_R) R)(\frac{1}{2} + \frac{b+d-d^2-r+r^2}{2\beta}) > (t_R R + (1-t_D) D)(\frac{1}{2} - \frac{b+d-d^2-r+r^2}{2\beta})
\]

or

\[
b > -(d - d^2 - r + r^2) - \frac{\beta((2t_D - 1)D - (2t_R - 1)R)}{D + R}
\]

Because \( b \) is symmetric around 0, the probability that party \( D \) wins is \( F(d - d^2 - r + r^2 + \frac{\beta((2t_D - 1)D - (2t_R - 1)R)}{D + R}) \). Aggregate voter turnout and the winning margin can be obtained by subtracting and adding the two sides of equation 3.

#### A.2 Proof of Theorem 2.

Assuming that parties have chosen policy platforms \( d \) and \( r \) in stage 1, the optimal amount of campaign spending in the second stage can be found and written as a function of \( d \) and \( r \). Let \( i = D, R \) denote each party and let \( p_i = d, r \) denote party \( i \)'s policy
choice and $E_i = D, R$ denote party $i$’s effort. Also, let $F_i$ denote $F$ if $i = D$ and $1 - F$ if $i = R$. The objective function of party $i$ is:

$$F_i((d - d^2 - r + r^2) - \frac{\beta(2t_D - 1)D - (2t_R - 1)R}{D + R})(-1 + p_i) - E_i$$

$$= F_i((d - d^2 - r + r^2) - \frac{\beta((2t_D - 1)D - (2t_R - 1)R)}{D + R})(1 - d - r + G) - 1 + p_i - E_i$$

If $1 - d - r + G \leq 0$, then the unique Nash equilibrium choice of campaign spending is $D = R = 0$ since both parties are either indifferent about winning or want the other party to win. When $1 - d - r + G > 0$, both parties will spend $E_i > 0$. Using the objective function above, the first order condition for spending in the second stage of the game for either party is:

$$1 \leq f((d - d^2 - r + r^2) - \frac{\beta((2t_D - 1)D - (2t_R - 1)R)}{D + R})(1 - d - r + G) - 1 + p_i - E_i$$

which is equivalent to

$$1 \leq (1 - d - r + G)\beta(2t_D + 2t_R - 2)E_i(D + R)^{-2}/2\alpha$$

with strict equality if $E_i < 1$. The unique solution to this system of equations is

$$D = R = \min\{\frac{\beta(t_D + t_R - 1)}{2}(1 - d - r + G)/2\alpha, 1\}$$

Since the second derivative of the objective function for either party nonpositive, this represents the unique Nash equilibrium of campaign spending given any $d, r$.

I now prove a series of Lemmas leading up to the proof of Theorem 3.

### A.3 Proof of Theorem 3

**Lemma 1.** Given any $p_{-i}$, party $i$’s best response policy choice is such that $1 - d - r + G \geq 0$

**Proof.** I focus on the problem solved by $D$ since the problem solved by party $R$ is entirely symmetry. As discussed in A.2, if $1 - d - r + G \leq 0$, the unique Nash equilibrium

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4The following section is an exact recreation of Herrera’s proof using the terms that were derived in the previous two sections.
for spending in the second stage is $D = R = 0$. The objective function of party D can be written as

$$F(d - d^2 - r + r^2)(1 - d - r + G) - 1 + r$$

over the interval $\{d : d \geq 1 - r + G\}$. The derivative of the objective function with respect to $d$ is

$$-F(d - d^2 - r + r^2) + \frac{1 - 2d}{2\alpha}(1 - d - r + G)$$

or equivalently

$$-(1/2 + (d - d^2 - r + r^2)/2\alpha) + \frac{1 - 2d}{2\alpha}(1 - d - r + G).$$

This expression is strictly negative if $1 - d - r + G < 0$ for any $d < 1/2$. If $d \geq 1/2$, this expression is strictly negative if

$$-4d + 3d^2 - r^2 + 1 + 2dr < 2\alpha - 1 + (2d - 1)G$$

or equivalently if

$$-4d + 4d^2 - (d - r)^2 < 2\alpha - 1 + (2d - 1)G,$$

which is verified since $d \leq 1$ and $\alpha \geq 1$.

**Lemma 2.** Given any $p_{-i} \leq 1/2$, party $i$’s best response policy choice is such that $p_i < 1/2$.

**Proof.** I focus on the problem solved by party D since the problem solved by party R is entirely symmetric. Using the previous lemma, we have that, given any policy choice $r$ by party R, the best response $d^*$ by party D is such that $1 - d - r + G \geq 0$. Using Theorem 2, Assumption 2, and $G \leq 1$, if $1 - d - r + G \geq 0$ then the unique second stage Nash choice of campaign spending is given by

$$D = R = \beta\left(\frac{t_D + t_R - 1}{2}\right)(1 - d - r + G)/2\alpha.$$

Thus, the objective function of party D in the first stage of the game, anticipating correctly the campaign spending choices of both parties, is

$$F(d - d^2 - r + r^2)(1 - d - r + G) - 1 + r - \beta\left(\frac{t_D + t_R - 1}{2}\right)(1 - d - r + G)/2\alpha. \quad (3)$$

The derivative of this objective function with respect to $d$ is

$$-F(d - d^2 - r + r^2) + \frac{1 - 2d}{2\alpha}(1 - d - r + G) + \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha. \quad (4)$$
Now, suppose that, given some policy choice \( r \leq 1/2 \) by party \( R \), the best response \( d^* \) by party \( D \) is such that \( d^* > 1/2 \). Using Eq. (4), the derivative of the objective function at \( d^* \) is nonnegative only if

\[
F(d^* - (d^*)^2 - r + r^2) - \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha < 0.
\]

Using Eq. (3), the objective function of party \( D \) evaluated at \( d^* \) is

\[
(F(d^* - (d^*)^2 - r + r^2) - \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha)(1 - d^* - r + G) - 1 + r.
\]

The first term in this expression is not positive since \( 1 - d^* - r + G > 0 \) and \( F(d^* - (d^*)^2 - r + r^2) - \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha < 0 \). Thus, party \( D \) is better off deviating to \( d = r \), because \( 1 - 2r + G > 0 \) and \( 1/2 - \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha > 0 \) (using Assumption 2).

**Lemma 3.** Given any \( p_i \leq 1/2 \), party \( i \)'s payoff is strictly concave in its own policy choice in the interval \([0, 1/2]\).

**Proof.** I focus on the problem solved by party \( D \) since the problem solved by party \( R \) is entirely symmetric. Suppose that \( r \in [0, 1/2] \), and consider the problem of party \( D \). For \( d \leq 1/2 \), we have \( 1 - d - r + G \geq 0 \). Thus, for \( d \leq 1/2 \), the second derivative of the objective function, as derived from Eq. (5), is

\[
-(1 - 2d)/\alpha - (1 - d - r + G)/\alpha < 0.
\]

**Lemma 4.** In equilibrium, \( d = r < 1/2 \).

**Proof.** Using Lemma 2, we have that in equilibrium \( d \neq 1/2 \) and \( r \neq 1/2 \). Now suppose that \( d = r > 1/2 \). Using Assumption 2 and Lemma 1, we can see that the derivative of the objective function of either party as given by Eq. (4) is negative, a contradiction.

Suppose \( d > r > 1/2 \) (the case of \( r > d > 1/2 \) is similar). Using the first order condition for either party, we obtain

\[
1 - d - r + G = \frac{F_i(d - d^2 - r + r^2) + \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha}{(1 - 2p_i)/2\alpha}
\]

for \( i = D, R \). Note that the left-hand side is independent of \( i \). Thus,

\[
\frac{F(d - d^2 - r + r^2) + \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha}{1 - F(d - d^2 - r + r^2) + \beta\left(\frac{t_D + t_R - 1}{2}\right)/2\alpha} = \frac{d - 1/2}{r - 1/2}.
\]

Since \( d > r \) and \( d + r > 1 \), we have that \( d - d^2 - r + r^2 < 0 \), which implies \( F(d - d^2 - r + r^2) < 1/2 \). Thus, the left-hand side is smaller than one. However, \( d > r \) implies that the right-hand side is larger than one, a contradiction.
Suppose $d < r < 1/2$ (the case $r < d < 1/2$ is similar). Then $d - d^2 - r + r^2 < 0$, which implies $F(d - d^2 - r + r^2) < 1/2$. Thus, the left-hand side of Eq. (5) is smaller than one. However, if $d < r$, then the right-hand side $((1/2-d)/(1/2-r))$ is larger than one, a contradiction.

**Proof of Theorem 3.** Lemmas 1-4 imply that in equilibrium $d = r = p^* < 1/2$, where (using Eq. (4)) $p^*$ satisfies the first order condition

$$-1/2 + (1 - 2p^*)(1 - 2p^* + G)/2\alpha + \beta \left(\frac{t_D + t_R - 1}{2}\right)/2\alpha = 0.$$  

Solving this quadratic equation I obtain the desired result.

**B Proof that D=R for the general case**

I now prove that given any values for each party’s targeting effectiveness and backfire rate, campaign spending in the second stage will be the same for both parties. Let $t_D$, $b_D$ and $t_R$, $b_R$ be the targeting effectiveness and backfire rate for parties D and R respectively. Then party D wins if

$$(t_D D + b_R R) \left( \frac{1}{2} + \frac{b + d - d^2 - r + r^2}{2\beta} \right) > (t_R R + b_D D) \left( \frac{1}{2} - \frac{b + d - d^2 - r + r^2}{2\beta} \right)$$

or equivalently

$$b < (d - d^2 - r + r^2) + \frac{\beta(t_D - b_D)D - (t_R - b_R)R}{(t_D + b_D)D + (t_R + b_R)R}.$$

Because $t_D$, $b_D$, $t_R$, and $b_R$ are constants, I can substitute a,b,c, and e for $t_D - b_D$, $t_R - b_R$, $t_D + b_D$, and $t_R + b_R$. Both parties attempt to maximize the objective function

$$F_i((d - d^2 - r + r^2) + \frac{\beta aD - bR}{cD + eR}) (1 - d - r + G) - 1 + p_{-i} - E_i$$

Assuming that $1 - d - r + G > 0$ and using the objective function above, the first order condition for spending in the second stage of the game is:

$$1 \leq f((d - d^2 - r + r^2) + \frac{\beta aD - bR}{cD + eR}) (1 - d - r + G) \beta (ae - bc) E_{-i} (cD + eR)^{-2}$$

which is equivalent to

$$1 \leq (1 - d - r + G) \beta (ae - bc) E_{-i} (cD + eR)^{-2}/2\alpha$$

with strict equality if $E_i < 1$. The solution to the series of equations is of the form $D = R$. 

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References


