Extreme Macroeconomic Events and Hedge Fund Performance: Identifying Hidden Risks in Quantitative Long/Short Equity and Risk Arbitrage

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Abstract

This paper considers hidden risks present within the hedge fund industry by focusing on two particular investment strategies. Quantitative long/short equity funds presented a new and interesting opportunity for risk assessment when many experienced massive losses over three days in August 2007. Following the approach of Khandani and Lo (2007), I employ a “contrarian” trading model to replicate these returns and to better understand systematic risk within the strategy. In addition to a successful “robustness check” on Khandani and Lo’s finding that the losses were most likely a result of the deleveraging of one or more large equity portfolios, I empirically justify a theory that contrarian returns are negatively related to market volatility, offer a new approach to estimating and analyzing leverage, and present a qualitative discussion of the hidden risks associated with portfolio overlap. Risk arbitrage, the second hedge fund strategy, is known to exhibit a non-linear and option-like risk and return profile. Utilizing the piecewise linear estimation model of Mitchell and Pulvino (2001), I find that returns of a particular merger arbitrage mutual fund and hedge fund index exhibit strong market correlation during down markets and little or no relationship during appreciating markets. Since this closely resembles a short position in an index put option, I adjust risk using a Black-Scholes replicating portfolio to find annual excess returns generated by risk arbitrage of about 4.5%. These conclusions, although useful in this context, cover just two hedge fund strategies. The intent is to emphasize the importance of alternative methods of risk assessment and to encourage further research into the topic of hidden risk.
1 Introduction

The hedge fund industry has enjoyed remarkable growth since the inception of the first hedge fund by Alfred W. Jones in 1949.\(^1\) Investors seeking additional sources of portfolio diversification unattainable within typical stock, bond, and mutual fund investments have found the unique risk and return profiles of hedge funds highly attractive. Assets under management were estimated at approximately $2.7 trillion as of 2007 and show few signs of decreasing.\(^2\) However, recent disruptions in the financial markets and the collapse of several prominent hedge funds highlight the substantial risk present within the industry and the need to further understand the nature of hedge fund investments.

The term *hedge fund* refers to a private investment fund with a particular legal organization and fee structure. A hedge fund is typically organized as a limited partnership, with the manager acting as general partner and the investors as limited partners. Restrictions on minimum investments, investor qualifications, and the marketing of funds typically limit the investor base to wealthy individuals and institutions but also allow for a light regulatory environment. Hedge funds are generally exempt from the various registration, leverage, and disclosure requirements applicable to mutual funds and broker-dealers.\(^3\) “Two and twenty” fees are most common, whereby investors are charged a 2% management fee and a 20% performance fee. Often, returns must meet a minimum threshold — commonly a *high water mark* or *hurdle rate* — before the performance fee applies.\(^4\) Though most hedge funds are organized similarly, the particular investment approach can vary widely from fund to fund.

Hedge funds are categorized based on several broad investment strategies. The Credit Suisse/Tremont Hedge Fund Index separates funds into ten general groups:

- Convertible Arbitrage

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\(^1\)See Rappeport (2007).


\(^3\)See Fung and Hsieh (1999a) for details.

\(^4\)See Liang (1999), Liang (2001), and Goetzmann et al. (2003).
- Dedicated Short Bias
- Emerging Markets
- Equity Market Neutral
- Event Driven (Distressed, Multi-Strategy, and Risk Arbitrage)
- Fixed Income Arbitrage
- Global Macro
- Long/Short Equity
- Managed Futures
- Multi-Strategy

Hedge Fund Research, Inc. offers a similar index family. Please see Appendix A for index definitions and strategy descriptions.

A key characteristic of many hedge fund strategies is a low correlation with the market. The flexibility provided by longer lockup periods and a light regulatory environment allow many managers to pursue investments that, by nature, are less cyclical and have less dependence on the financial markets and prevailing economic conditions. Consider private equity funds which make large, actively-managed investments in entire firms for relatively long periods of time. The risk and return profile of these funds is naturally more idiosyncratic than a typical stock investment. Activist hedge funds take very similar positions and, as a result, the distinction between private equity and hedge funds has blurred in recent years. Hedge funds are also better equipped to manage nontraditional and illiquid assets that may contain fewer elements of systematic risk.

Hedge funds also hedge. Expertise in short-selling and derivatives trading allow many funds to manage their portfolios in such a way so as to minimize systematic risk. The long/short equity and market neutral strategies trade on both the long and short sides
of the market, allowing the market-related components of individual securities to move in opposite directions. Since much of the systematic risk can be hedged away, managers are free to focus more on their area of expertise and less on the movements of the overall market. For example, one who is skilled at identifying mispriced stocks can capture the mispricing spreads without taking on a great deal of broad stock market exposure. Risk arbitrage, a strategy I analyze in Section 4, involves taking both long and short positions in firms involved in mergers and acquisitions in order to make a pure bet on whether the deal will succeed. In theory, a properly hedged position would be unaffected by market swings that affect both firms.

In contrast to the relative flexibility and autonomy enjoyed by the hedge fund industry, mutual funds are subject to tighter regulatory constraints, leading to separate methodologies for risk and performance evaluation. Mutual fund performance is typically measured against a broad market index corresponding to the fund’s particular strategy. An equity mutual fund might be tracked against the S&P 500 or the Russell 2000, whereas a bond fund might be compared to the Lehman Aggregate Bond Index or a high-yield or municipal bond index if appropriate. In any case, the objective is to maximize excess return over the benchmark while minimizing tracking error. A mutual fund that consistently “beats the market” generates substantial value for its investors because doing so requires superior security selection, sector weighting, and transaction cost minimization over time. Since added value comes in the form of outperformance, a 3% return when the market is up by 1% is more or less equivalent in the eyes of investors to a -3% return when the market is down by 5%. Mutual fund investors seek market exposure and pay the management fee in expection of excess returns over the relevant index. Performance measures such as the Sharpe ratio and Modigliani-Modigliani

\[ S = \frac{R_p - R_f}{\sigma_p} \]

The Sharpe ratio measures a portfolio’s excess return over the risk-free rate per unit of total risk.
(M-Squared)\(^6\) are useful for analyzing complete investment portfolios, while Jensen’s alpha\(^7\) and Treynor’s measure\(^8\) can be used to compare investment alternatives. Those based on relative return and systematic risk are useful for ranking mutual funds.

Hedge fund managers operate under quite separate objectives. The goal is to generate consistently positive absolute returns regardless of how the market performs. Naturally, this requires at least some mitigation of systematic risk because a positive return would be nearly impossible with high market correlation during a down market. The expectation is that returns should be primarily a function of the manager’s investment skill within his or her realm of expertise. Passive market exposure is available cheaply through index funds or exchange-traded funds (ETF’s), so a savvy investor would only pay the high fees for non-market return. This concept is generally referred to as pure alpha or portable alpha, as excess returns can be separated from market-derived returns through investments in hedge funds. The agile investor is then free to separately allocate between alpha and beta according to risk tolerance. In some cases, such as within a fund of funds or a large institutional operation, the investors may be sophisticated enough to manage systematic risk themselves. However, such investors would be unlikely to pay much for market exposure and would consequently demand higher excess returns and detailed information about the risk profile of the fund.

It is important for hedge funds to follow their stated investment strategy and risk profile

\(^6\)M^2 evaluates relative performance by first adjusting the portfolio’s risk to that of the market:

\[
M^2 = R_p^* - R_m
\]

where \( R_p^* = (1 - w)R_f + wR_p \)

\( w = \frac{\sigma_m}{\sigma_p} \)

\(^7\)Jensen’s alpha (commonly just alpha) is defined as portfolio return minus the risk-equivalent return estimated by the Capital Asset Pricing Model (CAPM). Thus,

\[
\alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]
\]

\(^8\)Treynor’s measure, in contrast to the Sharpe ratio, normalizes excess return by systematic risk:

\[
T = \frac{R_p - R_f}{\beta_p}
\]
in order to be consistent with their investors’ desired portfolio allocation. A drastic shift in strategy can leave investors over- or under-invested in particular asset classes. For example, when Jeffrey Vinik moved a large percentage of the Fidelity Magellan Fund into bonds, the historically large-cap growth fund assumed very different risk and return characteristics. The point is not that managers should be overly constrained to a particular approach but that it is important that investors accurately understand the risk profiles of their investments. Even more serious consequences arise when risk is misunderstood by fund managers. If assets move in unexpected ways in response to market events, the entire fund is in jeopardy because hedges may become ineffective or even risk-increasing. The hidden risks of both of these situations present a major problem for investors and warrant an in-depth analysis designed to quantify and explain them.

Given both the perceived and realized value added by hedge funds (low market correlation and positive absolute returns) and the particular strategies they follow to achieve these goals, a valid question to ask is: **How can one assess the total risk profile of a hedge fund strategy?** Other statistical measures such as the appraisal (or information) ratio\(^9\) are of even more relevance to the hedge fund industry and can often provide a reasonable assessment of performance in the context of risk-adjusted returns. It is well understood that if a fund’s systematic risk is higher than claimed, the risk-adjusted returns must be lower. However, complications such as asymmetric risk profiles, benchmark uncertainty, and little to no disclosure challenge the limits of the current models at our disposal and call for alternative approaches to risk assessment. Whereas the risk and return characteristics of mutual funds are widely available and well understood, hedge funds are not required to report results or portfolio characteristics, making risk assessment a non-trivial task. Even if this data were available, the complex and dynamic nature of the industry make risk difficult

\(^9\)The appraisal ratio can assist in developing an optimal portfolio allocation over separate funds. It measures alpha per unit of idiosyncratic risk:

\[
AR_p = \frac{\alpha_p}{\sigma(\epsilon_p)}
\]
This paper focuses on the risk and return analysis of two hedge fund strategies: quantitative long/short equity and risk arbitrage. Long/short equity hedge funds utilize various security analysis techniques to build both long and short positions and profit from both stock price appreciations and depreciations. In contrast, mutual funds are typically invested purely in the long side of the market due to regulatory restrictions. Note that the long/short strategy does not necessarily imply market neutrality — as is the case with equity market neutral funds — because a portfolio may be constructed to be long or short biased. A dedicated short bias fund has such a constraint; thus, long/short equity is the most general of the three strategies. The analysis in Section 3 focuses on the quantitative long/short equity strategy, where “quantitative” implies that trading decisions are primarily model-driven. This style is also referred to as “statistical arbitrage” because these funds attempt to exploit pricing inefficiencies among large groups of stocks. The second strategy is risk arbitrage which, as previously mentioned, involves betting on whether a merger or acquisition will succeed. Consider a typical stock merger where the acquirer buys a target firm using its own stock as payment. The risk arbitrageur would then buy the target firm’s stock and sell short the acquirer’s stock according to the offered ratio (for example, 1-for-1). The spread corresponds to the market’s probability assessment on whether or not the deal will succeed, and the investment earns this spread if it does. Note that neither of the two hedge fund strategies is necessarily market neutral but that the overall portfolios may be augmented with appropriate hedges to keep the fund’s beta arbitrarily low.

For the following study, I infer risk profiles using a representative strategy (for the quantitative long/short equity strategy) and mutual fund and hedge fund index returns (for risk arbitrage). These methods cannot hope to give exact risk measures for individual hedge funds but can provide reasonable estimates for the so-called “average hedge fund” within a particular strategy. Such an approach supplies tangible, numerical results with which one can make more general statements and raise further research questions. The inductive
transition from modeled results to generalizations parallels the imperfect transformation of
private knowledge into limited publicly available information. For this approach, I further
focus on hedge fund performance in the context of extreme macroeconomic events because
this type of analysis is ostensibly of greatest value to investors.

This paper is organized as follows. Section 2 provides a broad overview of existing hedge
fund literature with a focus on alternative means of risk and performance analysis. Sections 3
and 4 are “mini-chapters” on quantitative long/short equity and risk arbitrage, respectively.
I suggest additional paths for future research in Section 5 and conclude in Section 6.

2 Hedge Fund Literature

The hedge fund industry has inspired a great deal of academic literature. The unique risk and
return characteristics of hedge funds introduce an appealing new source of research questions
with which academics hope to gain a better understanding of alternative investments and the
field of investment management in general. Agarwal and Naik (2005) conveniently summarize
existing hedge fund literature in book form. Key topics include performance evaluation,
risk assessment, and asset allocation. Studies such as Fung and Hsieh (1999a) offer an
introduction to hedge fund industry structure and discuss several of the strategies listed in
Appendix A. One important industry characteristic concerns fee structure, particularly the
“high water mark” contractual feature that sets performance fees contingent upon continued
performance, which Goetzmann et al. (2003) and Liang (2001) analyze in great detail.

A recurring theme is the inadequacy of traditional mean-variance models in the context
of hedge fund analysis. The classic portfolio allocation problem, as presented by Markowitz
(1952), is to maximize expected returns while minimizing the variance of returns. Typically,
the Capital Asset Pricing Model (CAPM) or another factor model, such as the five-factor
Fama and French (1993) model, is used to project expected returns, and past historical
volatility serves to estimate future variance. Modern portfolio theory remains tremendously
powerful for the purposes of broad asset allocation decisions, but the difficulty in determining the true risk of hedge fund investments poses a significant complication. Although Fung and Hsieh (1999b) find that mean-variance analysis leads to approximately correct rankings of hedge funds, they admit that it is not appropriate for risk assessment.

One problem is the low correlation of hedge fund returns with traditional asset classes. As Fung and Hsieh (1997) show in the first published academic article on hedge funds, the indices commonly used to explain much of the variation of mutual fund returns tend to yield low $R^2$ values when applied to hedge fund returns. Agarwal and Naik (2000b) and Liang (2001) confirm that much of the variation in hedge fund returns remains unexplained by standard market indices. In the search for explanatory factors, Fung and Hsieh (1997) use a purely statistical technique to show that five principal components\(^{10}\) of hedge fund returns explain approximately 45% of the cross-sectional variation. In other words, common factors endogenous to the hedge fund industry possess much more explanatory power than the typical broad market indices. Therefore, much work has been done to maintain hedge fund indices that, by construction, serve as more useful benchmarks for risk and performance evaluation. The Credit Suisse/Tremont and Hedge Fund Research indices can be useful in this regard. Additionally, Fung and Hsieh (2002) suggest a hedge fund of funds index as a useful benchmark due to its more accurate representation of the true hedge fund investor experience.

Researchers have also used returns-based style analysis as introduced by Sharpe (1992) in an attempt to assess the exposure of hedge funds to various asset classes. This “reverse engineering” approach is particularly useful in the context of minimal disclosure because other approaches based on holdings or publicly available data are simply infeasible. Ben Dor et al. (2003) demonstrate the importance of including derivative benchmarks along with other style factors to account for the option-like returns that result from active management, as

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\(^{10}\)Principal component analysis involves reducing the dimensionality of large data sets through an orthogonal linear transformation. Primarily used for purposes of exploratory data analysis, PCA recovers the most predictive and variance-explaining components embedded in the data set.
shown by Glosten and Jagannathan (1994). To assist with accurate fund classification, Ben Dor et al. (2006) suggest reconciling the stated investment strategy of individual funds with results from style analysis.

Another issue involves the asymmetric and non-linear nature of hedge fund returns. Asness et al. (2001) demonstrate that betas tend to be underestimated due to factors such as the trading of illiquid securities, mark-to-model pricing, and window dressing. Fung and Hsieh (2001), Agarwal and Naik (2004), and Mencia (2006) discuss the presence of a non-linear relationship between hedge fund returns and market conditions, particularly in funds following market timing and event-driven strategies. As illustrated in Section 4, Mitchell and Pulvino (2001) have made considerable headway in quantifying this type of systematic risk.

Other statistical difficulties pose considerable problems for academics and investors alike. As Brooks and Kat (2001) point out, serial correlation, kurtosis, and skewness lead to an over-allocation of capital to hedge funds. Getmansky et al. (2004) use the serial correlation of returns to stress the considerable illiquidity present within many hedge funds.

A third difficulty concerns the biases present in many hedge fund databases. As discussed in Fung and Hsieh (2000), hedge fund databases contain three main types of biases. Selection bias arises from the regulatory implications of performance disclosure, namely that no such reporting is required. Furthermore, it is not clear which types of funds tend be included or omitted. One plausible proposition is that poorly performing hedge funds are less likely to report returns. However, an equally reasonable argument is that top-performing funds may choose not to disclose performance in order to keep profitable trading strategies from being replicated. The second bias is survivorship bias. The problem, which arises when failed funds are excluded from databases, is also applicable to mutual funds, as Brown et al. (1992) illustrate. To highlight the non-triviality of this complication, Liang (2000) finds a survivorship bias of over 2% per year in two of the leading hedge fund databases. Finally, instant history bias stems from the retroactive inclusion of historical returns as new funds are added to a database. Thus, it is a phenomenon of the actual hedge fund databases, not
the inherent structure of the industry. Naturally, such a bias is problematic as it can cause one to arrive at different conclusions depending on when the database is accessed.

Performance evaluation is perhaps the most often addressed issue in hedge fund academic literature. Alpha, in the original Jensen (1967) sense, is difficult to assess as it is based on a deterministic and linear risk model that is not appropriate for most hedge funds. Ackermann et al. (1999) find that hedge funds outperform mutual funds but experience much more volatility. Using alternative methods, Kosowski et al. (2007) and Fung and Hsieh (2004) show that hedge funds deliver significant alpha after adjusting for various risk factors. In a very interesting study concerning the technology bubble, Brunnermeier and Nagel (2004) find that hedge funds did not exert a correcting force on inflated asset prices as theorized by the efficient markets hypothesis, but instead profited from them and sold out just before the peak. The topic of performance persistence is important for distinguishing between manager skill and luck. Agarwal and Naik (2000a) find persistence to be particularly strong over short intervals while Jagannathan et al. (2008) find significant persistence among superior funds and little among inferior funds.

The study of poorly performing hedge funds arguably provides even more insight than that of successful funds. For every George Soros or John Paulson, there are many hedge fund managers that do not generate large returns or even succeed at all. Getmansky et al. (2004) analyze hedge fund failures and draw a number of conclusions leading to regulatory suggestions that promote hedge fund stability. Edwards (1999) analyzes the fall of Long-Term Capital Management, a subject I return to in Section 3.4. An excellent paper by Khandani and Lo (2007) analyzes the large losses experienced by many quantitative equity funds during August 2007. This event forms the basis of my analysis of the quantitative long/short equity strategy in Section 3.
3 Quantitative Long/Short Equity

3.1 Background

In this section, I examine the quantitative long/short equity strategy by constructing the returns of a theoretical equity portfolio based on a simple mechanical trading rule. The particular rule involves a bet on mean reversion, where the portfolio is long underperforming stocks and short outperforming stocks, as compared to the overall market. This “contrarian strategy” was formalized by Lo and MacKinlay (1990) and most recently used by Khandani and Lo (2007) to study the massive losses experienced by many quantitative equity funds in August 2007.

The contrarian model provides for an interesting analysis of risk within the quantitative long/short equity sector. The very nature of this trading strategy suggests a very low exposure to systematic risk, at least in the context of CAPM betas. Modern portfolio theory aside, one would still expect such funds to be largely uncorrelated with an equity index due to the offsetting long and short positions. A large degree of idiosyncratic risk is also unlikely due to the significant diversification benefits of investing in hundreds or thousands of different securities, as in a typical quantitative equity portfolio. The large losses of August 2007 are thus inconsistent with a conventional modeling approach and suggest the presence of hidden risk within this strategy.

It is worthwhile to recall the market environment during the summer months of 2007. In May of that year, Bear Stearns halted redemptions on two of its structured credit hedge funds and in July reported losses of 91% in the more conservative of the two funds.11 Later in July, the well-regarded Sowood Capital Management announced losses in excess of 50% and a forced sale of assets to Citadel Investment Group due to large declines in the value of its credit holdings.12 The U.S. sub-prime mortgage crisis was well under way and had soon spread into other credit markets, including commercial paper and auction-rate debt, and

11See Morgenson (2007).
12See Strasbury and Burton (2007).
across the world, including Chinese investment banks and Australian hedge funds. And then, on August 7, 8, and 9, several quantitative long/short equity hedge funds suffered record losses while the S&P 500 returned +0.62%, +1.41%, and -2.96%, respectively. On August 10, the Wall Street Journal reported that in August alone, a Renaissance Technologies fund had lost 8.7%, Highbridge Capital Management’s Statistical Opportunities and Market Neutral funds were down 18% and 5.2%, and Tykhe Capital’s largest fund had fallen by 20%.\textsuperscript{13} On August 13, the Goldman Sachs Global Equity Opportunities Fund disclosed that it had lost over 30% of its value during the previous week alone!\textsuperscript{14} These funds, along with several others that had also suffered, follow dynamic, model-driven trading strategies, presumably with very little pure exposure to the market. Even a hypothetical, highly-leveraged fund with a long or short bias could not have suffered large losses on all three days if market exposure were the only risk factor at play. The implication is that there must exist some source of intra-strategy correlation that caused this remarkable chain of losses, suggesting that an alternative characterization of risk is appropriate.

To model these events, I follow the approach of Khandani and Lo (2007) to construct daily returns of the contrarian strategy for several key reasons. First, the contrarian strategy, although simple and mechanical, is a viable proxy for the average quantitative long/short equity fund, as I demonstrate in Section 3.3. I also show the link between the theoretical returns and actual reported returns by making some intuitively reasonable assumptions about leverage. Second, contrarian trading is very easy to understand. Buying underperforming stocks and shorting outperformers is essentially a “reverse momentum” trade. Lower re-turning stocks in one period will earn higher returns in the next (and vice versa) if mean reversion occurs. Third, the simple trading strategy allows for a relatively straight-forward generation of the return time series and, more importantly, avoids the excessive deployment of unnecessarily constrictive assumptions. The goal is to make general inferrences and an overly elaborate model would only cloud the results. Finally, the contrarian strategy has

\textsuperscript{13} See Zuckerman et al. (2007).
\textsuperscript{14} See Thal Larsen (2007) and Sender et al. (2007).
been rigorously analyzed by the academic community, most notably by Lo and MacKinlay (1990), leading to a number of theoretical tools for analyzing the data.

To examine the expected returns of a bet on mean reversion, I use Lo and MacKinlay’s model of contrarian profitability. The authors show that expected returns can be separated into a) autocovariances, b) cross-autocovariances, and c) variance among individual stock returns. The directional effect of each term can be summarized as follows:

\[
E[\pi] = C - A - V
\]

where \(C\), \(A\), and \(V\) correspond to cross-autocovariance, autocovariance, and variance, with the signs indicative of the contribution of each component to expected return.\(^{15}\)

Market overreaction, the most common explanation of contrarian profitability, represents the autocovariance component of expected return. Negative autocorrelation, or the presence of price reversals, is consistent with profits, holding the other terms constant. The implication is that, on average, a return in excess of the mean in one period implies a return under the mean in the next. For example, a particularly strong earnings statement may stimulate a high demand for the stock, leading to further price appreciation due to herd behavior. Conversely, an adverse event such as a lawsuit or negative analyst report may induce investor fear and fuel an overselling of the stock. Presumably, the stock would revert toward its initial price after the overly positive or negative sentiment has receded.

Although this explanation is certainly valid and intuitively reasonable, Lo and MacKinlay have shown that the contrarian strategy can be profitable even in the absence of market overreaction. The profitability of the contrarian strategy persists even though there is substantial empirical evidence of positive autocorrelation in stock portfolios (for example, the S&P 500), an observation inconsistent with the overreaction explanation. Thus, contrarian profits amidst positive autocorrelation must be due to strong cross-autocorrelations between stocks because variance can never be negative. Although this observation is secondary to

\(^{15}\)See Appendix B for the derivation of Equation (3.1) from the trading model in Section 3.2.
the primary purpose of risk assessment, I include it primarily to provide further evidence that the contrarian strategy is a justifiable proxy for the quantitative long/short strategy.

In the remainder of Section 3, I present the contrarian model, follow the methods used by Khandani and Lo (2007) to build the contrarian series of returns, compare the contrarian strategy to an appropriate hedge fund index, analyze profitability amidst changes in stock market volatility, suggest a unique approach to leverage estimation, and offer a qualitative discussion of portfolio overlap and hidden risk.

3.2 Contrarian Model

I generate contrarian returns by following the simple portfolio weighting scheme first introduced by Lehmann (1990), further analyzed by Lo and MacKinlay (1990), and most recently used for the August 2007 analysis by Khandani and Lo (2007). Portfolio weights are set proportional to a stock’s performance relative to the market portfolio over the previous trading day. I recompute weights and returns daily to mimic the highly dynamic trading systems used by quantitative hedge funds. Specifically, a stock’s weight is the negative of the deviation of its latest daily return from the market return, normalized by the number of tradable stocks. Formally,

\[ w_{it} = -\frac{1}{N} (R_{it-1} - R_{mt-1}) \]  

(3.2)

where \( w_{it} \) denotes the weight of stock \( i \) at time \( t \) and \( R_{it-1} \) and \( R_{mt-1} \) are the lagged daily returns of the individual stock and the market portfolio, respectively, with the market portfolio defined as equally-weighted:

\[ R_{mt} = \frac{1}{N} \sum_{i=1}^{N} R_{it} \]  

(3.3)
Note that, by construction, the strategy is dollar-neutral:

\[
\sum_{i=1}^{N} w_{it} = -\frac{1}{N} \sum_{i=1}^{N} (R_{it-1} - R_{mt-1}) \tag{3.4}
\]

\[
= -\frac{1}{N} \left[ \sum_{i=1}^{N} R_{it-1} - \sum_{i=1}^{N} R_{mt-1} \right] \tag{3.5}
\]

\[
= -\frac{1}{N} (NR_{mt-1} - NR_{mt-1}) \tag{3.6}
\]

\[
= 0 \tag{3.7}
\]

Although dollar-neutrality does not necessarily imply market-neutrality, it is often a close approximation when betas are similarly distributed within the long and short portfolios.

Since net investment is zero, portfolio returns have no well-defined value in theory. However, in practice, the Federal Reserve’s Regulation T limits leverage for most accounts to 2:1, where leverage is defined as the sum of the absolute values of both long and short positions.\(^{16}\)

Therefore, such a leverage constraint implies a minimum net investment, \(I_t\), of

\[
I_t = \frac{1}{2} \sum_{i=1}^{N} |w_{it}| \tag{3.8}
\]

I now define the daily return, \(R_{pt}\), of the contrarian strategy as the weighted average stock return divided by net investment:

\[
R_{pt} = \frac{\sum_{i=1}^{N} w_{it} R_{it}}{I_t} \tag{3.9}
\]

### 3.3 Data Description

Daily stock returns data were obtained from the Center for Research in Security Prices (CRSP) for the period between January 1995 and December 2007. At the time of publication of Khandani and Lo (2007), year 2007 data was not yet available and was instead based on

\(^{16}\)See http://www.federalreserve.gov/Regulations for details.
historical prices from Yahoo! Finance. The ability to use CRSP data for the entire period avoids the mismatch problem between the two databases and allows for the incorporation of dividends into return calculations and thus can only improve the accuracy of the results. Like the previous study, I drop stocks as they fall outside of the $5 to $2,000 price range\textsuperscript{17}, change from CRSP share code 10 or 11\textsuperscript{18}, or cease trading altogether. Table 1 reports the average number of unique stocks traded by the contrarian strategy during each year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Stock Count</th>
</tr>
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<tbody>
<tr>
<td>1995</td>
<td>5104</td>
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<tr>
<td>1996</td>
<td>5600</td>
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<tr>
<td>1997</td>
<td>5714</td>
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<td>1998</td>
<td>5401</td>
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<td>1999</td>
<td>5023</td>
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<td>3899</td>
</tr>
<tr>
<td>2007</td>
<td>3855</td>
</tr>
</tbody>
</table>

Table 1: Average Number of Unique Stocks Traded by Contrarian Strategy

Figure 1 portrays the relationship between the contrarian returns (see Section 3.4) and the appropriate hedge fund index. The index is the Hedge Fund Research (HFR) Equity Market Neutral Index, as that is the most applicable to the contrarian strategy.\textsuperscript{19} In order to compare the two return series side by side, I normalize the monthly returns of each to unit variance to correct for differences in volatility. Note that the purpose is to illustrate the correlation between the two and not the absolute relationship. A simple regression of

\textsuperscript{17}Presumably, stocks outside of this range are more difficult and more costly to trade due to illiquidity.  
\textsuperscript{18}Share codes 10 and 11 identify U.S. common stocks.  
\textsuperscript{19}Credit Suisse/Tremont lists quantitative long/short as a sub-sector of the equity market neutral strategy. See Appendix A.
normalized contrarian returns on normalized index returns specified by

\[
\frac{R_{ct}}{\sigma_c} = \alpha + \beta \frac{R_{it}}{\sigma_i} + \varepsilon_t
\]  

(3.10)

gives a coefficient of 0.21, as reported in Table 2. Although the \( R^2 \) of the regression is small, the coefficient is significant at the 1% level, validating the use of the contrarian model as a reasonable proxy for the overall hedge fund strategy.

![Normalized Monthly Returns of Contrarian Strategy and HFR Equity Market Neutral Index](image)

Figure 1: Relationship Between Contrarian Strategy and HFR Equity Market Neutral Index

### 3.4 Results and Analysis

Portfolio weights and returns are constructed exactly according to the contrarian model. Average daily returns, standard deviations, and annualized Sharpe ratios are reported in Table 3. Although leverage is a crucial determinant of actual fund returns, I list the raw returns here and later adjust accordingly.
Index Regression Results

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>0.2088</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.0788)**</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1.0242</td>
</tr>
<tr>
<td>(0.0987)**</td>
<td></td>
</tr>
</tbody>
</table>

N 156

$R^2$ 0.0436

** denotes significance at 1% level

Table 2: Contrarian Returns Regressed on HFR Equity Market Neutral Index

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Daily Returns</th>
<th>Standard Deviation of Daily Returns</th>
<th>Sharpe Ratio $(R_f = 0%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>1.35%</td>
<td>0.41%</td>
<td>52.15</td>
</tr>
<tr>
<td>1996</td>
<td>1.15%</td>
<td>0.49%</td>
<td>37.19</td>
</tr>
<tr>
<td>1997</td>
<td>0.89%</td>
<td>0.68%</td>
<td>20.57</td>
</tr>
<tr>
<td>1998</td>
<td>0.52%</td>
<td>0.95%</td>
<td>8.70</td>
</tr>
<tr>
<td>1999</td>
<td>0.33%</td>
<td>1.11%</td>
<td>4.75</td>
</tr>
<tr>
<td>2000</td>
<td>0.35%</td>
<td>1.72%</td>
<td>3.22</td>
</tr>
<tr>
<td>2001</td>
<td>0.31%</td>
<td>1.38%</td>
<td>3.58</td>
</tr>
<tr>
<td>2002</td>
<td>0.45%</td>
<td>0.96%</td>
<td>7.38</td>
</tr>
<tr>
<td>2003</td>
<td>0.16%</td>
<td>0.54%</td>
<td>4.64</td>
</tr>
<tr>
<td>2004</td>
<td>0.36%</td>
<td>0.58%</td>
<td>9.77</td>
</tr>
<tr>
<td>2005</td>
<td>0.23%</td>
<td>0.45%</td>
<td>7.94</td>
</tr>
<tr>
<td>2006</td>
<td>0.13%</td>
<td>0.52%</td>
<td>4.06</td>
</tr>
<tr>
<td>2007</td>
<td>0.23%</td>
<td>0.74%</td>
<td>4.94</td>
</tr>
</tbody>
</table>

Table 3: Summary Statistics of Daily Contrarian Returns, By Year

The most notable observation to be made from Table 3 is the declining average return and Sharpe ratio of the contrarian trading strategy between 1995 and 2007. A standard explanation for this trend is increased competition within the quantitative long/short equity sector. As is the case with almost any profitable trading rule or market inefficiency, returns diminish over time as more and more market participants begin to follow the strategy. Khandani and Lo (2007) show using TASS hedge fund data that assets under management under long/short equity and equity market neutral funds have increased from under $10 billion in 1995 to over $160 billion in 2007. They also point out the significant growth of
similar strategies such as 130/30 (invested 130% long using proceeds from short sales of 30%). Although the contrarian strategy is just one possible quantitative trading method, it is conceivable that the statistical models used by the various funds have significant overlap. Such an overlap would create an interesting intra-strategy correlation that presents a new source of risk.

Another cause of the decline in contrarian profitability may be the increase in stock market volatility between 1995 and 2002. Figure 2 plots monthly contrarian returns against the average monthly value of the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), scaled down by a factor of 100. The spikes in implied volatility in 1998 (during the Russian debt and Long-Term Capital Management crisis) and in 2001 and 2002 (the September 11 terrorist attacks, collapse of the dot-com bubble, accounting scandals, and recession) were followed by significantly lower contrarian returns. Indeed, a regression of contrarian returns on the contemporaneous and one- and two-month lagged VIX Index yields a statistically significant coefficient of -0.57 on the twice-lagged implied volatility term. The finite distributed lag model is as follows:

\[
R_{ct} = \alpha + \beta_1 VIX_t + \beta_2 VIX_{t-1} + \beta_3 VIX_{t-2} + \varepsilon_t \quad (3.11)
\]

See Table 4 for full results. The empirical evidence is consistent with Equation (3.1), in that higher volatility leads to lower expected returns.

An important implication of declining returns in the quantitative long/short equity sector is a proportional increase in the need to employ leverage to maintain the high returns demanded by investors. Although precise figures of hedge fund leverage are unavailable, the mismatch between declining returns and growing competition suggest that it has increased. Empirically, the long/short equity and equity market neutral strategies showed fairly consistent gains even amidst the quickly growing amount of assets under management. The construction of the contrarian series of returns presents an opportunity to quantify this
Figure 2: Negative Correlation Between Volatility Spikes and Contrarian Profitability

Table 4: Contrarian Returns Regressed on the Contemporaneous and Lagged VIX Index

<table>
<thead>
<tr>
<th>VIX Regression Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX&lt;sub&gt;t&lt;/sub&gt;</td>
</tr>
<tr>
<td>(0.2321)</td>
</tr>
<tr>
<td>VIX&lt;sub&gt;t−1&lt;/sub&gt;</td>
</tr>
<tr>
<td>(0.3268)</td>
</tr>
<tr>
<td>VIX&lt;sub&gt;t−2&lt;/sub&gt;</td>
</tr>
<tr>
<td>(0.2324)*</td>
</tr>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>(0.0231)**</td>
</tr>
</tbody>
</table>

| N   | 154          |
| R<sup>2</sup> | 0.0603      |

* denotes significance at 5% level
** denotes significance at 1% level

trend.

A simple model to generate reasonable leverage estimates is to determine the leverage necessary to maintain 1995-level returns in subsequent years. As discussed in Section 3.2,
Regulation T implies a leverage ratio of 2:1, or two dollars invested per dollar of capital. I will assume that this value represents the initial leverage employed in 1995 and define the leverage multiple as total leverage normalized with respect to Regulation T, or, equivalently, current leverage divided by year 1995 leverage. The leverage multiple in any particular year is then lagged by one year to prevent forward looking bias. So, for example, year 2007 leverage is a function of the leverage multiple implied by year 2006 returns. Using this methodology, I construct leveraged daily returns of the contrarian strategy as shown in Table 5.

<table>
<thead>
<tr>
<th>Year</th>
<th>Raw Daily Returns</th>
<th>Implied Leverage Multiple</th>
<th>Leveraged Daily Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>1.35%</td>
<td>1.00</td>
<td>1.35%</td>
</tr>
<tr>
<td>1996</td>
<td>1.15%</td>
<td>1.00</td>
<td>1.15%</td>
</tr>
<tr>
<td>1997</td>
<td>0.89%</td>
<td>1.17</td>
<td>1.04%</td>
</tr>
<tr>
<td>1998</td>
<td>0.52%</td>
<td>1.52</td>
<td>0.79%</td>
</tr>
<tr>
<td>1999</td>
<td>0.33%</td>
<td>2.58</td>
<td>0.86%</td>
</tr>
<tr>
<td>2000</td>
<td>0.35%</td>
<td>4.05</td>
<td>1.42%</td>
</tr>
<tr>
<td>2001</td>
<td>0.31%</td>
<td>3.86</td>
<td>1.21%</td>
</tr>
<tr>
<td>2002</td>
<td>0.45%</td>
<td>4.30</td>
<td>1.92%</td>
</tr>
<tr>
<td>2003</td>
<td>0.16%</td>
<td>3.01</td>
<td>0.48%</td>
</tr>
<tr>
<td>2004</td>
<td>0.36%</td>
<td>8.52</td>
<td>3.06%</td>
</tr>
<tr>
<td>2005</td>
<td>0.23%</td>
<td>3.75</td>
<td>0.86%</td>
</tr>
<tr>
<td>2006</td>
<td>0.13%</td>
<td>5.91</td>
<td>0.79%</td>
</tr>
<tr>
<td>2007</td>
<td>0.23%</td>
<td>10.12</td>
<td>2.34%</td>
</tr>
</tbody>
</table>

Table 5: Construction of Leveraged Contrarian Returns Using Implied Leverage Multiples

As postulated, leverage has in fact increased considerably between 1995 and 2007. Since the leverage multiple is defined with respect to the 2:1 initial level, implied leverage in 2007 is approximately 20:1. This is not inconceivable considering various indications of hedge fund leverage in the financial press. Roger Lowenstein, in his book, *When Genius Failed: The Rise and Fall of Long-Term Capital Management*, reports that the giant fixed income arbitrage fund had a leverage ratio of about 30, magnifying a return on capital of only about 2.45% in 1995.²⁰

²⁰See Lowenstein (2000) for details.
I attempt to estimate actual leverage levels within quantitative funds by comparing the performance of the theoretical strategy with reported loss figures from Section 3.1. Although the model does not account for transaction costs and trading restrictions, the constructed returns are useful for analyzing trends and making general inferences. Indeed, the August 2007 results listed in Table 6 indicate a clear correspondence between the performance of the contrarian strategy and actual hedge funds during the week of August 6. This relationship suggests that the raw theoretical returns may serve as a useful approximation of the unleveraged returns of actual funds. Thus, one estimation technique is to apply constant leverage to the contrarian returns so as to equate the cumulative return over a particular period with actual reported performance numbers. The leverage multiple that forces this equality serves as a useful first approximation. Formally,

\[ R_{t_1:t_2}^* = \prod_{t=t_1}^{t_2} (1 + \theta R_{ct}) - 1 \] (3.12)

where \( \theta \) denotes the implied leverage multiple, \( R_{ct} \) is the contrarian return on day \( t \), and \( R_{t_1:t_2}^* \) represents the return reported by a fund over the period \( t_1 \) to \( t_2 \). One possible value for \( R_{t_1:t_2}^* \) is -30%, corresponding to the loss reported by the Goldman Sachs Global Equity Opportunities Fund during the week of August 6. Setting \( t_1 = 6 \) and \( t_2 = 10 \) (for August 6-10), solving for \( \theta \) gives a leverage multiple of 12.13. Another useful figure is the 18% month-to-date loss incurred by the Highbridge Statistical Opportunities Fund as of August 8. Contrarian returns would have to be scaled by 5.81 in order to yield this result over the six trading days. These two estimates suggest total leverage of about 24:1 and 12:1 in the two funds, which bound the previously suggested 20:1 level implied by Table 5. Since market frictions would no doubt lead to lower contrarian returns in practice, true leverage usage is potentially much higher than suggested.

Another estimation method involves considering the potential for a rapid reduction in leverage in response to large losses. As indicated by the financial press\(^{21}\) and discussed by

\(^{21}\)See Sender et al. (2007).
Khandani and Lo (2007), risk management controls almost certainly called for a fire sale of liquid stock positions in order to raise cash and to limit the magnitude of any further losses. Thus, the funds ostensibly ended the week considerably less leveraged than they had entered it. A simple modification to Equation (3.12) is to allow for two different leverage multiples:

$$R_{t_1:t_2} = \left[ \prod_{t=t_1}^{t_d-1} (1 + \theta_1 R_{ct}) \right] \left[ \prod_{t=t_d}^{t_2} (1 + \theta_2 R_{ct}) \right] - 1$$

(3.13)

assuming the fund altered its leverage from $\theta_1$ to $\theta_2$ on day $t_d$. A reasonable assumption is that the losses on August 7, 8, and 9 were primarily a function of initial leverage and that deleveraging was largely complete by the beginning of trading on August 10. Also, since the estimates assuming constant leverage are no longer appropriate, I will set $\theta_1 = 10.12$, as that is the implied leverage multiple of the contrarian strategy for 2007. Using the weekly loss of the Global Equity Opportunities Fund and setting $t_d$ to August 10, solving for $\theta_2$ gives a new leverage multiple of 7.46, a 26% decrease in leverage. Of course, the “back of the envelope” nature of these calculations is not at all rigorous and cannot hope to provide anything more than rough approximations. However, the estimates do offer a valuable insight into the otherwise difficult to quantify trend of rising leverage. Furthermore, actual returns are consistent with modeled returns in a relative sense, with leverage acting as a crucial determinant of loss potential.

Considering the high degree of leverage and the significant strategy overlap within the quantitative long/short equity realm, a very likely explanation for the August 2007 meltdown is that a sizeable deleveraging of one or more equity portfolios triggered a chain reaction of liquidations within funds holding similar positions.\footnote{This is the primary conclusion reached by Khandani and Lo (2007).} Whether or not the “portfolio unwind” occurred within a quantitative hedge fund or a larger proprietary trading desk is unknown, and perhaps unimportant, but the massive losses experienced amidst a relatively stable environment in the equity markets are consistent with this hypothesis. The credit markets, in
contrast, were in large part dysfunctional, with the fallout from the sub-prime mortgage crisis just beginning to be felt. Credit was being withdrawn, several markets had all but ceased trading, and the general sentiment had quickly turned to fear and distrust. Conceivably, the uncertainty in the credit markets was particularly worrisome to highly leveraged trading desks and hedge funds, and it is likely that risk controls unrelated to the equity markets called for a risk-reducing portfolio deleveraging. When large positions need to be unwound quickly, traders typically sell the most liquid securities, exerting downward pressure on prices. As prices drop, funds holding the same positions will see the need to reduce risk by cutting leverage, and so the cycle continues.

Table 6: Leveraged Returns of the Contrarian Strategy During August 2007
On August 10, the contrarian strategy returned an impressive 5.80%, likely the result of a large rebound in prices. However, actual funds had probably reduced leverage at just the most unfortunate time, transforming a geometrically compounded four-day loss of the contrarian strategy from -26% (assuming constant leverage) into a more than 33% decline in value (assuming a 26% deleveraging between August 9 and 10). One cannot fault the exercise of such risk controls because the future is never foreseeable. However, in hindsight, it is worth mentioning that forced deleveragings almost always occur at precisely the most inopportune times. Again consider the collapse of Long-Term Capital Management. It is widely known that most of the fixed income arbitrage positions were profitable in the long run, but the Russian debt crisis and subsequent widening of credit spreads forced a similarly unfortunate unwind, ultimately leading to the downfall of the hedge fund and a consortium-led bail-out of its portfolios.

These findings suggest a new source of hidden risk within the quantitative long/short equity sector: significant portfolio overlap, with its adverse effects amplified by substantial leverage. The growing presence of funds following similar trading strategies combined with a corresponding increase in trading volume on the same side of trades necessarily implies a higher than expected correlation among the hedge funds. This, I claim, is an important yet often neglected component of systematic risk. This type of risk is not merely a phenomenon of the quantitative long/short equity strategy but could very well exist in any number of hedge funds or large trading groups. An unexpected and unfavorable movement within any commonly held, highly leveraged portfolio could trigger a similar unwinding of positions with the potential for a chain reaction of liquidations resulting in massive losses.
4 Risk Arbitrage

4.1 Background

As briefly outlined in Section 1, risk arbitrage, or merger arbitrage, is an event-driven investment strategy designed to profit on the “arbitrage spread” associated with an impending merger or acquisition. A potential acquiring firm will typically make a bid with a higher value than the pre-offer stock price of the target company. Although this spread tends to shrink substantially immediately following the proposed deal’s announcement (or even beforehand if speculation or insider trading is involved), the two prices will generally not converge entirely. The remaining discrepancy represents the profit to be made by a risk arbitrageur should the deal succeed. However, it also represents the risk in risk arbitrage. Proposed deals can fail for any number of reasons including disagreements between parties, regulatory objections, and adverse conditions in the financial markets. For example, when Microsoft withdrew its bid for Yahoo! in May 2007 as a result of price disagreements, Yahoo! shares plunged by 15% in a single day.

There are two primary types of risk arbitrage investments. The first is applicable to cash mergers, where the target firm is offered cash for assets and/or equity. The arbitrageur simply purchases the stock of the target firm with the expectation of receiving the offered price should the deal go through. Returns are defined as in any other stock investment:

\[ R_{0,k} = \frac{P_T^k + \sum_{t=0}^{k} D_t^T}{P_0^T} - P_0^T \] (4.1)

where \( T \) denotes the target firm and \( t = 0 \) to \( t = k \) represents the holding period. The more complicated scenario occurs when the acquirer offers to buy the target firm with its own stock. In this case, arbitrage involves buying the target stock as before but additionally shorting the acquiring firm’s stock. As discussed in Section 1 in the context of hedging, the trade is a bet on spread convergence rather than absolute price. The proposed ratio
of acquirer shares to target shares determines Δ, the appropriate hedge ratio. Investment returns now include short sale proceeds invested at the risk-free rate, payment of acquirer dividends, and coverage of the short position at time $k$:

$$R_{0,k} = \frac{P^T_k + \sum_{t=0}^{k} D^T_t - P^T_0 - \Delta \left[ P^A_k + \sum_{t=0}^{k} D^A_t - P^A_0 \right]}{I_0} (1 + R_f)$$

where $A$ denotes the acquiring firm and $I_0$ is the initial investment. Although potential margin calls can significantly affect the size and timing of the base investment, a simple starting point is again with Regulation T initial margins:

$$I_0 = \frac{1}{2} \left( P^T_0 + \Delta P^A_0 \right)$$

In contrast to the theoretical method employed for the quantitative long/short equity analysis, I use actual returns to assess the risk and return characteristics of risk arbitrage. I have identified a mutual fund — the Merger Fund (ticker: MERFX) — and a hedge fund index — the Hedge Fund Research Merger Arbitrage Index$^{23}$ — that offer two independent sources of risk arbitrage returns data. The mutual fund is unique among three others following the merger arbitrage strategy in that it has existed for long enough to provide historical returns throughout varying market conditions, yielding statistically significant regression coefficients.$^{24}$ According to its prospectus,

The Fund invests at least 80% of its assets principally in the equity securities of companies which are involved in publicly announced mergers, takeovers, tender offers, leveraged buyouts, spin-offs, liquidations and other corporate reorganizations.

$^{23}$See Appendix A for index definitions.

$^{24}$The inception dates of the Arbitrage Fund (ARBFX) and the Enterprise Mergers & Acquisitions Fund (EMAXX) are in 2000 and 2001, respectively. Although the Gabelli ABC Fund (GABCX) was established in 1993, it follows many other strategies in addition to merger arbitrage, so its returns are not representative of the strategy.
The discussion of risk also seems appropriate:

The principal risk associated with the Fund’s merger-arbitrage investment strategy is that certain of the proposed reorganizations in which the Fund invests may be renegotiated or terminated, in which case losses may be realized.\(^{25}\)

The hedge fund index serves as a valuable “sanity check” of the results suggested by mutual fund returns. As a well-defined index, there is no question as to its relevance to the actual risk and return profile of the industry. Furthermore, it offers cross-sectionally smoothed data, conveniently reducing the effect of outliers on the estimation results.

It turns out that the assessment of risk within the risk arbitrage strategy is a non-trivial task. As discussed in Section 2, it has been well-established that traditional mean-variance models are often inadequate in the context of hedge fund analysis. Mitchell and Pulvino (2001) have focused specifically on risk arbitrage and indeed show returns to be much more positively correlated with declining markets than with appreciating markets. Accordingly, the strategy is stable during normal conditions, generating small alpha contributions when the market is appreciating or moving sideways. However, the large down-market beta implies negative returns during a falling market.

Note the analogy between this particular risk profile and that of writing put options on an equity index. Just as the option premium yields small and positive profits under normal conditions, risk arbitrageurs earn steady returns as long as the majority of deals remain successful. During a down market, the exercise of a put option causes losses for the writer just as the increased probability of deal failure can cause arbitrage spreads to widen and ultimately fall apart. Glosten and Jagannathan (1994) and Jagannathan and Korajczyk (1986) discuss the similarities between fund performance and option strategies.

Throughout the remainder of this section, I focus on quantifying the systematic risk present in risk arbitrage. I utilize the general econometric methods of Mitchell and Pulvino (2001) to examine the nature of the non-linear risk and return profile of the strategy. The use

\(^{25}\)See [http://www.fasttrack.net/ProspectusPageDisplay.asp?Sym=MERFX](http://www.fasttrack.net/ProspectusPageDisplay.asp?Sym=MERFX) for the full prospectus.
of actual returns offers a test of robustness on their approach based on the construction of two representative portfolios. My primary goal is to perform out-of-sample tests of Mitchell and Pulvino’s non-linear risk model and to examine other characteristics of the risk arbitrage strategy. I also revisit the option analogy to assess the profitability of risk arbitrage.

4.2 Estimation Model

As a starting point, I estimate the CAPM over the complete sample using both mutual fund and hedge fund index returns:

\[
R_{it} - R_{ft} = \alpha + \beta (R_{mt} - R_{ft}) + \varepsilon_t
\]  

(4.4)

To get a sense of the non-linearity of beta, I follow a similar approach to Mitchell and Pulvino by first limiting the sample to distinct market environments. I create several different samples using thresholds defined by the excess return of the market over the risk-free rate.

To justify the proposed similarity between risk arbitrage and the sale of index put options, I employ a piecewise linear model as specified in Mitchell and Pulvino (2001). The key is to identify a threshold, above which \( \beta \) is low and below which \( \beta \) is high, such that maximum explanatory power is obtained. I use a threshold of -4%, as this is the value that maximizes \( R^2 \) in the limited sample regressions, but also include values of -2% and 0% for completeness. The linear piecewise regression equation is as follows:

\[
R_{it} - R_{ft} = \delta [\alpha_{high} + \beta_{high} (R_{mt} - R_{ft})] + (1 - \delta) [\alpha_{low} + \beta_{low} (R_{mt} - R_{ft})] + \varepsilon_t
\]

\[
\delta = \begin{cases} 
1 & R_{mt} > T \\
0 & R_{mt} \leq T 
\end{cases}
\]  

(4.5)

where \( T \) is the threshold of excess market return and \( \alpha_{high}, \beta_{high} \) and \( \alpha_{low}, \beta_{low} \) denote the separate high-market and low-market alphas and betas. In order to ensure continuity —
that is, that the piecewise estimates are equal at the threshold — I impose the following constraint:

$$\alpha_{\text{high}} + \beta_{\text{high}} T = \alpha_{\text{low}} + \beta_{\text{low}} T$$  \hspace{1cm} (4.6)

### 4.3 Data Description

As discussed in Section 4.1, I use returns of the Merger Fund (MERFX) and the Hedge Fund Research Merger Arbitrage Index (HFR) as inputs to the estimation models. I obtain monthly returns of the mutual fund from the CRSP Mutual Fund Database for January 1989 (the fund’s inception date) through June 2007. I augment the 2007 data with manually calculated returns for July through December of 2007 based on historical prices and dividends reported by Yahoo! Finance. Monthly index returns are provided by Hedge Fund Research\textsuperscript{26} for January 1990 to December 2007. The market index is the CRSP value-weighted market portfolio while the risk-free rate is the ask yield on the one-month Treasury bill.


### 4.4 Results and Analysis

The CAPM regression results using limited samples are listed in Table 7. The significant non-linearity of systematic risk is highlighted by the beta coefficients associated with the most extreme thresholds. For example, MERFX has a beta not statistically different from zero in months where excess market returns are over 3% and a beta of 0.83 when the market is under the -6% threshold. The HFR index exhibits similar behavior, albeit with expectedly less variation. The limited sample betas are shown in graphical form in Figure 5. The left side of the graph corresponds to samples with upper thresholds ($R_m - R_f \leq T$) while the right side represents lower thresholds ($R_m - R_f \geq T$). This depiction further highlights the

\textsuperscript{26}Historical returns are available at http://www.hedgefundresearch.com.
Figure 3: Monthly Returns of MERFX Since Inception

Figure 4: Monthly Returns of Merger Arbitrage Index Since Inception
asymmetric nature of the up and down market betas.

Turning to the more descriptive piecewise linear model, I find that similar results hold. Tables 8 and 9 list the estimates for MERFX and the HFR index at the 0%, -2%, and -4% market excess return threshold levels. Note that the -4% threshold corresponds to the limited sample regression yielding the highest $R^2$ (see Table 7). Using $T = -4\%$, $\beta_{\text{high}} = 0.13$ and $\beta_{\text{low}} = 0.68$ for MERFX and $\beta_{\text{high}} = 0.09$ and $\beta_{\text{low}} = 0.37$ for the hedge fund index. Also, the null hypothesis that the coefficients are equal to zero can easily be rejected. Similar results hold at higher thresholds, with less correlation present as market returns increase. At 0%, the $\beta_{\text{high}}$ estimates are actually insignificant at all standard levels, suggesting no correlation between risk arbitrage returns and market returns when market excess returns are strictly positive. The equality of $\alpha_{\text{high}}$ and $\alpha_{\text{low}}$ in the 0% case is a result of the continuity constraint.

Figure 5: Limited Sample Betas at Various Upper and Lower Thresholds

<table>
<thead>
<tr>
<th>Maximum or Minimum Market Excess Return</th>
<th>MERFX</th>
<th>HFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.06</td>
<td>.8</td>
<td></td>
</tr>
<tr>
<td>-0.04</td>
<td>.6</td>
<td></td>
</tr>
<tr>
<td>-0.02</td>
<td>.4</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>.0</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>.0</td>
<td></td>
</tr>
</tbody>
</table>
Upper Thresholds \((R_m - R_f \leq T)\)

Lower Thresholds \((R_m - R_f \geq T)\)

<table>
<thead>
<tr>
<th>MERFX</th>
<th>α</th>
<th>0.0097</th>
<th>0.0098</th>
<th>0.0109</th>
<th>0.0121</th>
<th>0.0135</th>
<th>0.0148</th>
<th>0.0162</th>
<th>0.0177</th>
<th>0.0194</th>
<th>0.0212</th>
<th>0.0231</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>0.0155</td>
<td>0.0154</td>
<td>0.0153</td>
<td>0.0152</td>
<td>0.0152</td>
<td>0.0151</td>
<td>0.0150</td>
<td>0.0150</td>
<td>0.0149</td>
<td>0.0149</td>
<td>0.0148</td>
</tr>
<tr>
<td></td>
<td>(\rho)</td>
<td>0.0167</td>
<td>0.0169</td>
<td>0.0171</td>
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* denotes significance at 5% level
** denotes significance at 1% level

Table 7: CAPM Regression Results Using Limited Samples
Similar to Mitchell and Pulvino (2001) and Ben Dor et al. (2003), I generate scatter plots of excess merger arbitrage returns versus excess market returns with the kinked piecewise regression line superimposed. MERFX (Figure 6) exhibits higher betas than the hedge fund index in both up and down markets, a reflection of its status as an individual fund. The similar shape of the regression line for the hedge fund index (Figure 7) is an important result. The non-linear risk characteristics first discussed qualitatively and subsequently shown to be true for a particular mutual fund are indeed applicable to the average risk arbitrage hedge fund in the industry.

Just as traditional CAPM beta estimation is inappropriate for assessing systematic risk,
Figure 6: Merger Arbitrage Mutual Fund Returns versus the Market

Figure 7: Merger Arbitrage Hedge Fund Index Returns versus the Market
Jensen’s alpha is an inadequate performance measure. To estimate the true excess returns generated by risk arbitrage, I follow the contingent claims approach to performance evaluation as introduced by Glosten and Jagannathan (1994) and applied by Mitchell and Pulvino (2001). As discussed in Section 4.1 and verified in Section 4.4, risk arbitrage returns mimic those of selling uncovered index put options. Therefore, I calculate the value of a portfolio containing risk-free bonds and shorted put options that replicates the payoff of a risk arbitrage fund and then compare it to the amount initially invested in risk arbitrage. The difference is the true alpha, the amount of excess return generated by risk arbitrage.

Consider a one month $100 investment in risk arbitrage. To replicate the payoff, one would write $\beta_{low}$ put options with a strike price equal to $100 \left(1 + T + R_f\right)$ and invest in Treasury bills with a face value of $100 \left(1 + R_f + \alpha_{high}\right)$. Using a -4% threshold and the 1982-2007 average one-month Treasury yield of 41 basis points, the strike is set at $96.41. The Black-Scholes price for a put option on a $100 index at this strike, assuming a risk free rate of 4.92% (the annualized one-month Treasury rate), market volatility of 14.89% (annualized sample average), and one month to expiration, is $P = 0.3974$.

I take the values of $\alpha_{high}$ and $\beta_{low}$ from both the a) mutual fund estimation (0.0036 and 0.6757, respectively) and b) hedge fund index estimation (0.0053 and 0.3662) to construct the two positions. The mutual fund coefficients suggest a bond position of $100.77$ augmented with 0.6757 short puts, while the hedge fund coefficients suggest investing $100.94$ in bonds and writing 0.3662 put options. The cost of a replicating portfolio is simply the present value of the face value of the bond minus the received option premiums:

\[(a) \quad \text{Cost} = \frac{100.77}{1.0041} - 0.6757 (0.3974) = 100.09\]

\[(b) \quad \text{Cost} = \frac{100.94}{1.0041} - 0.3662 (0.3974) = 100.38\]

Thus, according to the contingent claims analysis, the Merger Fund generates 9 basis points
of excess return per month, or 1.08% annually, while the hedge fund index provides 38 basis points per month, or 4.56% annually. These figures are actually in line with the CAPM estimates in Table 7. Over the complete sample, the estimate for MERFX is 7 basis points per month (not statistically significant) and alpha for the HFR index is 40 basis points for month (significant at the 1% level). Although this may not always occur, the similarity here provides a good test of robustness to the contingent claims evaluation.

As a final note, I refer once more to the Mitchell and Pulvino (2001) risk arbitrage analysis. The authors estimate somewhat different high-market alphas and low-market betas based on their “value-weighted average return series” and “risk arbitrage index manager returns,” two approaches to theoretical portfolio construction. Using an identical contingent claims approach, they compute annual excess returns generated by risk arbitrage of about 4.0%. This is quite close to the 4.56% figure reported above for the hedge fund index. Even with differences in assumptions and data sources, both analyses present a very similar risk and return profile of risk arbitrage, verifying the non-linear model particularly relevant to the strategy.

5 Further Research

The first suggestion for further research, of relevance to both parts of this study, is the application of alternative models of representative portfolio generation. The contrarian approach, although extremely descriptive, is just one possible strategy, particularly in the model-driven field of quantitative investing. Other methods may include alternative weighting schemes depending on market value or fundamental analysis, asymmetric holding periods, or a greater focus on technical analysis using queues such as moving averages.

Methods used for the risk arbitrage analysis are admittedly basic, perhaps not in the modeling phase but certainly in terms of data analysis. I have described the merits of using mutual fund and hedge fund index returns as viable proxies for actual hedge fund returns,
but a deeper analysis may reveal biases similar to or distinct from those outlined by Fung and Hsieh (2000) and Liang (2000) when using hedge fund databases.

Finally, I encourage the ongoing development of risk measures related to other hedge fund strategies. Of particular use would be a characterization of the risk taken on by fixed income funds such as within convertible arbitrage. Also, although human factors such as those present in the Amaranth Advisors implosion are difficult to assess, the empirical analysis of risk in the Managed Futures realm would certainly add value to the overall field.

6 Conclusion

I have assessed the hidden risks associated with two separate hedge fund strategies: quantitative long/short equity and risk arbitrage. As first discussed, hedge funds provide an important source of diversification for many sophisticated investors seeking a source of return with low systematic risk. Many hedge funds claim to generate considerable alpha, but the verification of such claims can be a difficult task given the unique regulatory and structural hurdles present in the industry. However, as I and many others have shown, meaningful approximations can be made to this end. Specifically, alternative risk appraisals distinct from standard volatility and correlation-driven measures arising from modern portfolio theory often carry more descriptive power — and provide more meaningful characterizations of otherwise misunderstood risk — but are naturally more difficult to apply. The identification of intra-industry systematic risk present within quantitative strategies, arising from portfolio overlap and high degrees of leverage, and the non-linear degree of market correlation found in risk arbitrage are just two such results obtained through unconventional approaches to risk assessment. The call for new and innovative methods of analysis will no doubt increase as more and more capital flows into the industry and new sources of hidden risk inevitably arise.
A Hedge Fund Strategy Descriptions

The Credit Suisse/Tremont Hedge Fund Indices include a broad asset-weighted index as well as a total of thirteen sub-indices organized by strategy. The index descriptions are as follows:

**Convertible Arbitrage** funds aim to profit from the purchase of convertible securities and the subsequent shorting of the corresponding stock when there is a pricing error made in the conversion factor of the security. Managers typically build long positions of convertible and other equity hybrid securities and then hedge the equity component of the long securities positions by shorting the underlying stock or options. The number of shares sold short usually reflects a delta neutral or market neutral ratio. As a result, under normal market conditions, the arbitrageur generally expects the combined position to be insensitive to fluctuations in the price of the underlying stock.

**Dedicated Short Bias** funds take more short positions than long positions and earn returns by maintaining net short exposure in long and short equities. Detailed individual company research typically forms the core alpha generation driver of dedicated short bias managers, and a focus on companies with weak cash flow generation is common. To affect the short sale, the manager borrows the stock from a counter-party and sells it in the market. Short positions are sometimes implemented by selling forward. Risk management consists of offsetting long positions and stop-loss strategies.

**Emerging Markets** funds invest in currencies, debt instruments, equities and other instruments of countries with emerging or developing markets (typically measured by GDP per capita). Such countries are considered to be in a transitional phase between developing and developed status. Examples of emerging markets include China, India, Latin America, much of Southeast Asia, parts of Eastern Europe, and parts of Africa. There are a number of sub-sectors, including arbitrage, credit and event driven, fixed income bias, and equity bias.
Equity Market Neutral funds take both long and short positions in stocks while minimizing exposure to the systematic risk of the market (i.e., a beta of zero is desired). Funds seek to exploit investment opportunities unique to a specific group of stocks, while maintaining a neutral exposure to broad groups of stocks defined for example by sector, industry, market capitalization, country, or region. There are a number of subsectors including statistical arbitrage, quantitative long/short, fundamental long/short and index arbitrage. Managers often apply leverage to enhance returns.

Event Driven funds invest in various asset classes and seek to profit from potential mispricing of securities related to a specific corporate or market event. Such events can include: mergers, bankruptcies, financial or operational stress, restructurings, asset sales, recapitalizations, spin-offs, litigation, regulatory and legislative changes as well as other types of corporate events. Event Driven funds can invest in equities, fixed income instruments (investment grade, high yield, bank debt, convertible debt and distressed), options and various other derivatives. Many managers use a combination of strategies and adjust exposures based on the opportunity sets in each sub-sector.

Distressed Event Driven funds invest across the capital structure of companies subject to financial or operational distress or bankruptcy proceedings. Such securities trade at substantial discounts to intrinsic value due to difficulties in assessing their proper value, lack of research coverage, or an inability of traditional investors to continue holding them. This strategy is generally long-biased in nature, but managers may take outright long, hedged or outright short positions. Distressed managers typically attempt to profit on the issuers ability to improve its operation or the success of the bankruptcy process that ultimately leads to an exit strategy.

Multi-Strategy Event Driven managers typically invest in a combination of event driven equities and credit. Within the equity space, sub-strategies include risk arbitrage, holding company arbitrage, equity special situations, and value equities
with a hard or soft catalyst. Within the credit-oriented portion, sub-strategies include long/short high yield credit (sub-investment grade corporate bonds), leveraged loans (bank debt, mezzanine, or self-originated loans), capital structure arbitrage (debt vs. debt or debt vs. equity), and distressed debt (workout situations or bankruptcies) including post-reorganization equity. Multi Strategy Event Driven managers have the flexibility to pursue event investing across different asset classes and take advantage of shifts in economic cycles.

**Risk Arbitrage Event Driven** hedge funds attempt to capture the spreads in merger or acquisition transactions involving public companies after the terms of the transaction have been announced. The spread is the difference between the transaction bid and the trading price. Typically, the target stock trades at a discount to the bid in order to account for the risk of the transaction not closing successfully. In a cash deal, the manager will typically purchase the stock of the target and tender it for the offer price at closing. In a fixed exchange ratio stock merger, one would go long the target stock and short the acquirers stock according to the merger ratio, in order to isolate the spread and hedge out market risk. The principal risk is deal risk, should the deal fail to close.

**Fixed Income Arbitrage** funds attempt to generate profits by exploiting inefficiencies and price anomalies between related fixed income securities. Funds limit volatility by hedging out exposure to the market and interest rate risk. Strategies include leveraging long and short positions in similar fixed income securities that are related either mathematically or economically. The sector includes credit yield curve relative value trading involving interest rate swaps, government securities and futures; volatility trading involving options; and mortgage-backed securities arbitrage (the mortgage-backed market is primarily US-based and over-the-counter).
Global Macro funds focus on identifying extreme price valuations and leverage is often applied on the anticipated price movements in equity, currency, interest rate and commodity markets. Managers typically employ a top-down global approach to concentrate on forecasting how political trends and global macroeconomic events affect the valuation of financial instruments. Profits are made by correctly anticipating price movements in global markets and having the flexibility to use a broad investment mandate, with the ability to hold positions in practically any market with any instrument. These approaches may be systematic trend following models, or discretionary.

Long/Short Equity funds invest on both long and short sides of equity markets, generally focusing on diversifying or hedging across particular sectors, regions or market capitalizations. Managers have the flexibility to shift from value to growth; small to medium to large capitalization stocks; and net long to net short. Managers can also trade equity futures and options as well as equity-related securities and debt or build portfolios that are more concentrated than traditional long-only equity funds.

Managed Futures funds (often referred to as CTAs or Commodity Trading Advisors) focus on investing in listed bond, equity, commodity futures and currency markets, globally. Managers tend to employ systematic trading programs that largely rely upon historical price data and market trends. A significant amount of leverage is employed since the strategy involves the use of futures contracts. CTAs do not have a particular bias towards being net long or net short any particular market.

Multi-Strategy funds are characterized by their ability to allocate capital based on perceived opportunities among several hedge fund strategies. Through the diversification of capital, managers seek to deliver consistently positive returns regardless of the directional movement in equity, interest rate or currency markets. The added diversification benefits reduce the risk profile and help to smooth returns, reduce volatility and decrease asset-class and single-strategy risks. Strategies adopted in a multi-strategy fund
may include, but are not limited to, convertible bond arbitrage, equity long/short, statistical arbitrage and merger arbitrage.

Hedge Fund Research, Inc. offers a similar family of indices. Funds are classified over four broad investment styles, each containing several sub-indices based on strategy or sector focus. The organization of the HFR indices is summarized in Table 10. Strategy details are omitted as they are very similar to the Credit Suisse/Tremont definitions.27

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27See http://www.hedgefundresearch.com for full definitions.
## Strategy Classifications

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Table 10: Hedge Fund Research Hedge Fund Indices
B Derivation of Contrarian Strategy Expected Returns

Equation (3.1) gives a useful interpretation of the expected returns of the contrarian strategy in terms of a) autocovariances, b) cross-autocovariances, and c) variance among individual stock returns. The following is its derivation from Equation (3.9), which defines the realized daily return of the contrarian strategy. Although I alter the notation slightly to complement the qualitative discussion in Section 3.1, the algebra follows directly from Lo and MacKinlay (1990).

Daily contrarian returns are given by

\[ R_{pt} = \frac{\sum_{i=1}^{N} w_{it} R_{it}}{I_t} \]  

(B.1)

Thus, the daily profit per dollar invested is

\[ \pi_t = \sum_{i=1}^{N} w_{it} R_{it} \]  

(B.2)

Taking expected values, applying definitions, and rearranging the terms,

\[ E[\pi_t] = E \left[ \sum_{i=1}^{N} w_{it} R_{it} \right] \]  

(B.3)

\[ = E \left[ \sum_{i=1}^{N} -\frac{1}{N} (R_{it-1} - R_{mt-1}) R_{it} \right] \]  

(B.4)

\[ = -\frac{1}{N} \sum_{i=1}^{N} E[R_{it-1}R_{it}] + \frac{1}{N} E \left[ \sum_{i=1}^{N} R_{mt-1}R_{it} \right] \]  

(B.5)

\[ = -\frac{1}{N} \sum_{i=1}^{N} E[R_{it-1}R_{it}] + E[R_{mt-1}R_{mt}] \]  

(B.6)

\[ = -\frac{1}{N} \sum_{i=1}^{N} \left( \text{Cov}[R_{it-1}, R_{it}] + \mu_i^2 \right) + \left( \text{Cov}[R_{mt-1}R_{mt}] + \mu_m^2 \right) \]  

(B.7)

The covariance expressions represent the first-order autocovariances of stock i and the market, respectively. If written in terms of the first-order autocovariance matrix \( \Gamma_1 \) of the \( N \)
stocks, the expression becomes

\[
E[\pi_t] = -\frac{1}{N} \text{tr} (\Gamma_1) - \frac{1}{N} \sum_{i=1}^{N} \mu_i^2 + \frac{i' \Gamma_1 i}{N^2} + \mu_m^2 \tag{B.8}
\]

\[
= \frac{i' \Gamma_1 i}{N^2} - \frac{1}{N} \text{tr} (\Gamma_1) - \frac{1}{N} \sum_{i=1}^{N} (\mu_i - \mu_m)^2 \tag{B.9}
\]

The first term includes the off-diagonals, or cross-autocovariances, while the trace operator sums the elements on the matrix diagonal, providing a total of all \( N \) autocovariances. The third term is simply the variance among mean stock returns. Separating out the three effects with more algebra,

\[
E[\pi_t] = \frac{i' \Gamma_1 i}{N^2} - \left[ \frac{1}{N^2} \text{tr} (\Gamma_1) \right] - \frac{1}{N} \text{tr} (\Gamma_1) + \left[ \frac{1}{N^2} \text{tr} (\Gamma_1) \right] - \sigma^2 (\mu) \tag{B.10}
\]

\[
= \left[ \frac{1}{N^2} [i' \Gamma_1 i - \text{tr} (\Gamma_1)] \right] - \left[ \left( \frac{N-1}{N^2} \right) \text{tr} (\Gamma_1) \right] - \sigma^2 (\mu) \tag{B.11}
\]

\[
= C_1 - A_1 - V \tag{B.12}
\]

where \( C_1 \) and \( A_1 \) represent the cross-autocovariance and autocovariance terms and \( V \) denotes the variance. Finally, with the understanding that *daily* portfolio rebalancing implies *first-order* cross-autocovariances and autocovariances, I drop the subscripts to complete the derivation.

\[
E[\pi] = C - A - V \tag{B.13}
\]
Acknowledgements

I would like to thank Professor Ravi Jagannathan, Kellogg School of Management, for his exceptional guidance and support throughout the process of writing this paper. During our weekly discussions, he exposed me to the fascinating field of hedge fund research and developed my interest in a number of new and exciting topics in investment management. I am also grateful to Professor Todd Pulvino, Kellogg School of Management, for the helpful suggestions regarding merger arbitrage mutual funds that led to many of the results in Section 4.

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Third, I am very appreciative of the constructive feedback provided by Katie Wright, Michael Wright, Susan Whitcher, and John Whitcher. Their edits, suggestions, and additional perspectives were quite useful during the final stages of this project.

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