The Effects of Fairness and Equality on Employment Levels

in a Two Firm/Two Worker Type Model

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Senior Thesis
Mathematical Methods in the Social Sciences
May 27, 1988
ABSTRACT

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Some of the most recent work in the field of labor economics looks at the important effect of fairness on worker effort. This model uses optimization calculus and draws from certain concepts of efficiency wage models and the fair wage/effort hypothesis in an effort to explain the effects of fairness and equality on employment levels for a two firm/two worker type case. Given the fair wage and effort equations of the model, firms will pay both types of labor the same wage, and wages will be equal across firms. The result is that both labor markets cannot clear; the labor type with the lower market clearing wage will experience involuntary unemployment.
I. Introduction

The following excerpt from a textbook on personnel management illustrates the importance of equity to workers:

The need for equity is perhaps the most important factor in determining pay rates. . . . Externally, pay must compare favorably with those in other organizations or you'll find it hard to attract and retain qualified employees. Pay rates must also be equitable internally in that each employee should view his or her pay as equitable given other employees' pay rates in the organization (Dessler, 1984, pg. 223).

For a long time the field of labor economics viewed workers as something akin to capital, lacking human qualities. Pay was simply the price of labor - based on labor's marginal productivity - and productivity was inherent in the worker, independent of pay. Recently, we have seen a movement toward a more complex view of labor, recognizing that workers have human qualities which set them apart from machines. This, in turn, has lead to a focus on the effort provided by a worker. The purpose of this paper is to present a model which uses optimization calculus and draws on certain concepts of efficiency wage models and the fair wage/effort hypothesis in order to describe the effects of fairness and equity on employment levels. The model is of a simple two firm/two worker type case. Some background on the topic is presented in order to locate the model in an historical framework.

II. Neoclassical and Keynesian Models

Standard neoclassical models predict that workers will be paid their marginal products by cost-minimizing firms who purchase their services in competitive labor markets. A problem arises from these models: though most markets clear, this is not true of the labor market. We observe cyclical and apparently involuntary unemployment, which should theoretically not
exist if wages and employment levels were solely determined by factors of supply and demand.

Keynes was among the first to try to explain involuntary unemployment through the existence of "sticky" wages (Keynes, 1936). Sticky wages do not fall quickly enough to clear the labor market when demand is low. Also, low aggregate demand will lead to lower output and increases in unemployment. With this answer arises the inevitable question of why wages should be sticky. Do sticky wages reflect rational economic behavior? Attempts to answer these questions are found in contract theory and search theory.

III. Efficiency Wage Models

Recently, there has been developed a class of models called efficiency wage models which serve not only to explain cyclically varying involuntary unemployment but also to provide explanations for other labor phenomenon. In its most general form, the efficiency wage hypothesis holds that the services employers receive from their workers are a positive function of the wage they offer. The literature tends to focus on two labor market issues: the mechanisms whereby higher wages elicit higher productivity, and the implications of this relationship for the existence of involuntary unemployment at equilibrium. If wage cuts harm productivity, then cutting wages may end up raising labor costs. In equilibrium an individual firm's production costs are reduced if it pays a wage in excess of market clearing; thus, equilibrium involuntary unemployment results.

The concept of efficiency wages was first introduced in the context of underdeveloped countries, mainly emphasizing the relationship between wages and nutrition and effort (Leibenstein, 1957). A second area of efficiency wage theory is shirking models. The assumption here is that
payment of wages above market clearing, as well as the existence of a pool of unemployed workers, provides incentive for employees to work rather than to shirk (Foster and Wan, 1984; Miyazaki, 1984; Shapiro and Stiglitz, 1984). These models are especially relevant in cases where monitoring of production is costly, or otherwise prohibitive. Other models are concerned with the reduction of costly worker turnover (Salop, 1979; Stiglitz, 1974). These models are based on arguments similar to those used in shirking models. Finally, a last group of models deals mainly with issues of worker selection. The argument here is that, if reservation wages and ability are correlated, firms with higher wages should attract workers of higher ability (Malcomson, 1981; Weiss, 1980); thus, firms will not hire workers who offer to work for a lower wage than the firm's efficiency wage.

Although the main purpose of efficiency wage models is to explain involuntary unemployment, they provide some secondary results which explain such phenomenon as wage rigidity, dual labor markets, and wage distributions for workers of equal ability. Wage rigidity is the most obvious result of these models since the optimal action taken by the firm when facing decreased demand is to lay off workers, rather than to cut wages. Dual labor markets develop when wage-productivity relationships differ in importance across jobs. This phenomenon is closely related to the works on adverse selection (Malcomson, 1981; Weiss, 1980). The primary sector is designated as that sector in which efficiency wages are relevant. In this sector firms will pay wages in excess of market clearing, and unemployment will exist. The secondary sector acts according to neoclassical economic theory and experiences full employment. Finally, efficiency wage models serve to explain wage distributions for workers of equal potential. Since the wage-productivity relation may differ across firms, each firm, in theory, can
have a different efficiency wage, even though the workers themselves have the same characteristics with respect to effort.

To illustrate the relevance of efficiency wages in explaining involuntary unemployment, one might look at a rudimentary model. Consider an economy consisting of identical competitive firms, each facing the production function,

\[ q = f\left( e(w)n \right), \]

where \( e \) is the effort per worker, \( w \) is the wage, and \( n \) is the number of workers. A profit maximizing firm able to hire all the labor it wants at whatever wage it chooses will pay a wage \( w^* \) which satisfies the "Solow condition" (Solow, 1979): the elasticity of effort with respect to wage is unity (consult Appendix 1 for detailed explanation). The wage \( w^* \) is called the efficiency wage, and it minimizes cost per unit of effort. Each firm will hire workers up to the point where the marginal product of labor, \( e(w^*)f\left( e(w^*)n \right) \), equals the wage \( w^* \). If the total demand for labor falls below the total supply and \( w^* \) is greater than labor's reservation wage, the firms will be unaffected by labor market conditions in pursuing optimal policy, and, in equilibrium, involuntary unemployment will exist. Although unemployed workers would strictly prefer to be employed at the wage \( w^* \) rather than to be unemployed, firms will not hire them at that wage, or lower, because reducing the wage paid would lead to lower productivity for all employees already on the job.

IV. The Fair Wage/Effort Hypothesis

Some of the most recent work in the area of labor economics is a
modification of efficiency wage models. George Akerlof and Janet Yellen have developed the fair wage/effort hypothesis (July 1987), which is based on the assumption that "...workers have a conception of a fair wage; insofar as the actual wage is less than the fair wage workers supply a corresponding fraction of normal effort (pg. 1)." Furthermore, Akerlof and Yellen write that the underlying motivation for the hypothesis is, "When people do not get what they deserve, they try to get even (pg. 1)." The hypothesis is also supported by equity theory in psychology and social exchange and reference group theory in sociology. Both common sense and textbooks on personnel management acknowledge the importance of fairness to workers. The hypothesis serves to explain involuntary unemployment, as well as wage compression and unemployment/skill correlations.

According to the fair wage/effort hypothesis, the effort of a given type of labor is given by the equation

\[ e = \min \left( \frac{w}{w^*}, 1 \right), \]

where \( w \) is the actual wage paid and \( w^* \) is the perceived fair wage. One of the models developed in their paper considers the case of two worker types. It concludes that the group with the lower wage rate will experience unemployment. In the model, the fair wage for a particular type of labor has two determinants: the wage received by the other group of workers and the market clearing wage for this group of workers. The fair wage is the weighted average of these two components:

\[ w_1^* = \beta w_2 + (1-\beta)w_1^C \]
\[ w_2^* = \beta w_1 + (1-\beta)w_2^C \]
Using a production function given by a quadratic form in the effective labor power (effort per worker times number of workers) of both types of labor, Akerlof and Yellen derive labor demand functions for the two labor types. They use these functions to derive equations for the market clearing wages:

\[
\begin{align*}
\w^c_1 &= w_1 - (L_1 - L_1)/b_1 \\
\w^c_2 &= w_2 - (L_2 - L_2)/c_2 
\end{align*}
\]

Note that the equations contain an unemployment component; this term later introduces the need for unemployment in one of the sectors. At this point, Akerlof and Yellen have enough equations to solve the system. They show that unemployment is necessary in one of the sectors to lower this group’s perception of its fair wage; otherwise, the workers would deem it fair to receive the wage that the other group is receiving. (For specific equations, see Appendix 2A.)

Although the model provides important explanations, some of its concepts and procedures are questionable. The main problem is Akerlof and Yellen’s use of the market clearing wage. Since they only model a one firm case, the meaning of "market clearing" is vague; they provide no market to clear. They define the market clearing wage as "... that wage which, with the other wage held constant, is just enough lower to induce the hiring of the total labor supply [of type 1 or type 2 workers] respectively (pg. 24)." The labor demand equations derived from maximizing the firm's profit function are used to find the market clearing wage. In this respect, their work becomes a bit circular.

One can replicate the analysis used by Akerlof and Yellen in a model involving more than one firm. The result of a two firm analysis is that it is
possible for there to exist a low-paying firm and a high-paying firm. Furthermore, in such a situation, the existence of a low-paying firm will help curb the level of unemployment predicted by the one firm model. (For clarification, consult Appendix 2B.) One might conclude that, though the need for unemployment will not disappear, it may be more relevant to look at a closed model, i.e. where a specific number of firms comprise the industry, rather than employing the use of a vague "market" clearing wage.

V. The Model

The motivation behind developing the following model was to address the problems with the use of a market clearing wage while still incorporating the effects of equity and fairness on effort. Though this work draws heavily on the fair wage/effort hypothesis, my interest in this area preceded my reading of Akerlof and Yellen's July 1987 paper. I developed a model of wage determination which considered the importance of equity to workers (Flores, 1987). In this model of a two-person work group, wage dispersion serves as a disincentive term in the production function. The resulting optimal wages equal a constant plus the weighted average of a person's reservation wage and the wage received by the other person in the work group. These results provide support for the choice of fair wage equations used in the current model.

As stated previously, this model draws on some of the concepts presented by Akerlof and Yellen. The major difference is that this is a closed model, consisting of two firms which employ two types of labor. The effect of closing the model is to change the equations that determine fair wages. Consider the case of firms A and B employing labor types 1 and 2. Denote wages by $w_{ij}$, fair wages by $w_{ij}^*$, effort by $e_{ij}$, and number of workers
employed by $L_{ij}$, where $i$ denotes the type of worker and $j$ denotes the firm.

The fair wage equations depend on two components: the wage that a type of labor could receive at the other firm and the wage paid to the other type of workers within one's own firm. Letting the fair wage be the weighted average of these two components, the equations are as follows:

\[
\begin{align*}
    w_{1A}^* &= \beta_A w_{2A} + (1-\beta_A)w_{1B} \\
    w_{2A}^* &= \beta_A w_{2A} + (1-\beta_A)w_{2B} \\
    w_{1B}^* &= \beta_B w_{1B} + (1-\beta_B)w_{1A} \\
    w_{2B}^* &= \beta_B w_{2B} + (1-\beta_B)w_{2A}
\end{align*}
\]

Although the weights $\beta_j$ above are firm specific, it should be noted that making them type specific, firm and type specific, or constant across firms and types does not change the model's findings in any substantial way.

The effort of workers will depend on a comparison of the actual wage to the fair wage, provided the actual wage is less than the fair wage. It is assumed that effort equals one when the actual wage is equal to the fair wage and that effort cannot exceed this value even when the actual wage is greater than the fair wage. The effort equation for type 1 workers in firm $A$ is as follows:

\[
\begin{align*}
    e_{1A} &= e\left(w_{1A}/w_{1A}^*\right), \quad \text{for } w_{1A} < w_{1A}^* \\
    e_{1A} &= 1, \quad \text{for } w_{1A} \geq w_{1A}^*
\end{align*}
\]

Similar equations hold for $e_{2A}, e_{1B},$ and $e_{2B}$. The constraint on the shape of
the effort function is that it meet the Solow condition. In Appendix 3A, it is shown that this condition is indeed met. Because of this, we know that the firm will always pay the fair wage. We designate effort to equal one at this point because it simplifies the analysis.

The production functions for the two firms are given by a quadratic form in the effective labor power:

\[ Q_A = A_0 + A_1(e_1A^L_1A) + A_2(e_2A^L_2A) - A_{11}(e_1A^L_1A)^2 + A_{12}(e_1A^L_1A)(e_2A^L_2A) - A_{22}(e_2A^L_2A)^2 \]  

\[ Q_B = B_0 + B_1(e_1B^L_1B) + B_2(e_2B^L_2B) - B_{11}(e_1B^L_1B)^2 + B_{12}(e_1B^L_1B)(e_2B^L_2B) - B_{22}(e_2B^L_2B)^2 \]  

The profit functions for the two firms are given by the difference between output and costs, where the only costs are assumed to be the wages paid:

\[ \Pi_A = Q_A - w_1A^L_1A - w_2A^L_2A \]  

\[ \Pi_B = Q_B - w_1B^L_1B - w_2B^L_2B \]  

The profit functions are used to verify that the effort functions meet the Solow condition (see Appendix 3A) and to derive labor demand equations. Labor demand is found by taking the partial derivatives of the profit functions with respect to number of workers and solving for the number of workers. The demand functions are as follows:
\( L_{1A} = a_{1A} - b_{1A}w_{1A} + c_{1A}w_{2A} \) (9)

\( L_{2A} = a_{2A} + b_{2A}w_{1A} - c_{2A}w_{2A} \) (10)

where

\[ a_{1A} = (2A_{1}A_{22} - A_{2}A_{12})/(4A_{11}A_{22} - A_{12}^2) \]
\[ b_{1A} = (2A_{22})/(4A_{11}A_{22} - A_{12}^2) \]
\[ c_{1A} = (-A_{12})/(4A_{11}A_{22} - A_{12}^2) \]
\[ a_{2A} = (2A_{11}A_{2} + A_{1}A_{12})/(4A_{11}A_{22} - A_{12}^2) \]
\[ b_{2A} = (-A_{12})/(4A_{11}A_{22} - A_{12}^2) \]
\[ c_{2A} = (2A_{11})/(4A_{11}A_{22} - A_{12}^2) \]

\( L_{1B} = a_{1B} - b_{1B}w_{1B} + c_{1B}w_{2B} \) (11)

\( L_{2B} = a_{2B} + b_{2B}w_{1B} - c_{2B}w_{2B} \) (12)

where

\[ a_{1B} = (2B_{1}B_{22} - B_{2}B_{12})/(4B_{11}B_{22} - B_{12}^2) \]
\[ b_{1B} = (2B_{22})/(4B_{11}B_{22} - B_{12}^2) \]
\[ c_{1B} = (-B_{12})/(4B_{11}B_{22} - B_{12}^2) \]
\[ a_{2B} = (2B_{11}B_{2} + B_{1}B_{12})/(4B_{11}B_{22} - B_{12}^2) \]
\[ b_{2B} = (-B_{12})/(4B_{11}B_{22} - B_{12}^2) \]
\[ c_{2B} = (2B_{11})/(4B_{11}B_{22} - B_{12}^2) \]

\[ L_1 = L_{1A} + L_{1B} \]
\[ = (a_{1A} + a_{1B}) - (b_{1A}w_{1A} + b_{1B}w_{1B}) + (c_{1A}w_{2A} + c_{1B}w_{2B}) \] (13)

\[ L_2 = L_{2A} + L_{2B} \]
\[ = (a_{2A} + a_{2B}) + (b_{2A}w_{1A} + b_{2B}w_{1B}) - (c_{2A}w_{2A} + c_{2B}w_{2B}) \] (14)
The next step in the analysis is to examine the fair wage equations in order to determine what wages will be paid. Noting that the firms will pay the fair wage, equations (1) through (4) can be solved with wage $w$ equal to the fair wage $w^*$. We find the only possible solution is for all wages to be equal (Appendix 3B). This means that both types of labor will be paid the same wage and that wages will be equal across firms.

From this solution, it becomes apparent that both labor markets cannot clear in this model. Because the firms must pay both types of labor the same wage, it will be the case that the type of labor with the higher market clearing wage will experience full employment, while the other type of labor will experience unemployment. One of the types of labor will be paid a wage in excess of market clearing, and firms will hire less than the available number of this type. Firms will have no incentive to hire more of these workers at a lower wage because lowering the wage will lead to lower effort, in effect increasing labor costs. This situation can best be understood by looking at Figure 1. Assuming that type 2 labor has the lower market clearing wage, only $L_2$ workers will be hired, rather than the full $L_2$ workers available.

![Figure 1](image-url)
VI. Discussion and Conclusion

The preceding model explains the importance of fairness and equality on wage and employment decisions in a two firm/two worker type case. The result of the fair wage equations is that the firms will pay the same wage to both types of labor and that wages will be equal across firms. Because of this, the labor type with the lower market clearing wage will experience involuntary unemployment. The model also provides an explanation for the existence of wage compression and skill/unemployment correlations.

The greatest problem with this model is the strength of its findings in terms of equal wages for all workers in all firms. Although this solution follows from the given effort and fair wage equations, it is not a realistic situation. A more desirable result would be a case in which different types of labor are paid differing wages, though the wages for each type are equal across firms. Ideally, the wage distribution would be more compressed than it would be if wages were solely dependent on productivity due to the importance of equality and fairness. Further work should be done to determine whether increasing the number of labor types might result in a more realistic model.
The Solow condition is common to efficiency wage models. Assume the following profit function:

\[ P = f(e(w)n) - wn, \]  \hspace{1cm} (1)

where \( e \) is effort per worker, \( w \) is the wage, and \( n \) is the number of workers.

Take the partial derivatives of the profit function with respect to the wage and the number of workers and set them equal to zero.

\[ \frac{\partial P}{\partial w} = [f'(e(w)n)e'(w)n] - n = 0 \]  \hspace{1cm} (2)

\[ \frac{\partial P}{\partial n} = [f'(e(w)n)e(w)] - w = 0 \]  \hspace{1cm} (3)

Solving equations (2) and (3) gives us the Solow condition:

\[ e'(w) = e(w)/w \]  \hspace{1cm} (4)

To minimize costs per efficiency unit, the firm will choose a wage \( w^* \) such that the marginal cost of effort equals the average cost of effort.

![Figure 1-1](image-url)
APPENDIX 2A

The following are equations used by Akerlof and Yellen in the fair wage/effort model for a one firm and two worker types case:

I. Production and profit function

\[ Q = A_0 + A_1(e_1L_1) + A_2(e_2L_2) - A_{11}(e_1L_1)^2 + A_{12}(e_1L_1)(e_2L_2) - A_{22}(e_2L_2)^2 \]

\[ P = Q - w_1L_1 - w_2L_2 \]

II. Fair wage equations

\[ w_1^* = \beta w_2 + (1-\beta)w_1^C \]

\[ w_2^* = \beta w_1 + (1-\beta)w_2^C \]

III. Labor demand equations, with \( e_1 = e_2 = 1 \)

\[ L_1 = a_1 - b_1w_1 + c_1w_2 \]

\[ L_2 = a_2 + b_2w_1 - c_2w_2 \]

IV. Market clearing wage equations, derived from above labor demand eq'ns

\[ w_1^C = w_1 - (L_1 - L_1)/b_1 \]

\[ w_2^C = w_2 - (L_2 - L_2)/c_2 \]

Assume \( w_1 > w_2 \). In this situation type 2 workers will experience some unemployment because, without unemployment, they would deem it fair to receive the wage \( w_1 \). We can see this by solving for \( w_2^* \), substituting equation (8) into equation (4):

\[ w_2 = w_2^* = \beta w_1 + (1-\beta)[w_2 - (L_2 - L_2)/c_2] \]

\[ = w_1 - [(1-\beta)/\beta c_2] (L_2 - L_2) \]
APPENDIX 2B

One can replicate Akerlof and Yellen's analysis for a two-firm case. Briefly, the following equations would hold:

I. Production and Profit equations

\[ Q_A = A_0 + A_1(e_{1A-L1A}) + A_2(e_{2A-L2A}) - A_{11}(e_{1A-L1A})^2 + A_{12}(e_{1A-L1A})(e_{2A-L2A}) - A_{22}(e_{2A-L2A})^2 \]  

\[ Q_B = B_0 + B_1(e_{1B-L1B}) + B_2(e_{2B-L2B}) - B_{11}(e_{1B-L1B})^2 + B_{12}(e_{1B-L1B})(e_{2B-L2B}) - B_{22}(e_{2B-L2B})^2 \]  

\[ P_A = Q_A - w_{1A-L1A} - w_{2A-L2A} \]  

\[ P_B = Q_B - w_{1B-L1B} - w_{2B-L2B} \]  

II. Fair wage equations

\[ w_{1A}^* = B_A w_{2A} + (1-B_A)w_1 \]  

\[ w_{2A}^* = B_A w_{1A} + (1-B_A)w_2 \]  

\[ w_{1B}^* = B_B w_{2B} + (1-B_B)w_1 \]  

\[ w_{2B}^* = B_B w_{1B} + (1-B_B)w_2 \]  

III. Labor demand, using profit functions (3) and (4)

\[ L_{1A} = a_{1A} - b_{1A}w_{1A} + c_{1A}w_{2A} \]  

\[ L_{2A} = a_{2A} + b_{2A}w_{1A} - c_{2A}w_{2A} \]  

\[ L_{1B} = a_{1B} - b_{1B}w_{1B} + c_{1B}w_{2B} \]  

\[ L_{2B} = a_{2B} + b_{2B}w_{1B} - c_{2B}w_{2B} \]  

\[ L_1 = L_{1A} + L_{1B} \]  

\[ = (a_{1A} + a_{1B}) - (b_{1A}w_{1A} + b_{1B}w_{1B}) + (c_{1A}w_{2A} + c_{1B}w_{2B}) \]
\[ L_2 = L_{2A} + L_{2B} \]
\[ = (a_{2A} + a_{2B}) + (b_{2A}w_{1A} + b_{2B}w_{1B}) - (c_{2A}w_{2A} + c_{2B}w_{2B}) \]  

IV. Market clearing wage equations, derived using equations (13) and (14)

\[ w_1^C = \frac{b_1A}{(b_1A + b_1B)}w_{1A} + \frac{b_1B}{(b_1A + b_1B)}w_{1B} \]  
\[ - (L_1 - L_1)/(b_1A + b_1B) \]
\[ w_2^C = \frac{c_2A}{(c_2A + c_2B)}w_{2A} + \frac{c_2B}{(c_2A + c_2B)}w_{2B} \]  
\[ - (L_2 - L_2)/(c_2A + c_2B) \]

The above equations are sufficient to demonstrate the effects of having two firms in the model. Take the case where \( w_1 > w_2 \). In this case type 2 workers will experience unemployment. We can see this mathematically by substituting equation (16) into equation (6):

\[ w_{2A}^* = w_{2A} = d_A w_{1A} + (1-d_A)w_{2B} - \]
\[ [(1-\beta_A)/(c_2B + \beta_A c_2A)](L_2 - L_2) \]

where \( d_A = \beta_A(c_2A + c_2B)/(c_2B + \beta_A c_2A) \)

The important thing to note is that, although the wage equation still contains an unemployment term, the existence of the \( w_{2B} \) term may help to curb the level of unemployment, assuming that wages are not equal across firms for each worker type. The possibility of a high-paying firm and a low-paying firm may change Akerlof and Yellen's results. It would be interesting to see the model's conclusions in the case of a multi-firm industry. Then, at least the market clearing wage might have a better definition.
APPENDIX 3A

In order to derive the Solow condition for this model we need only to look at simplified versions of the production functions, rather than the actual functions given in equations (5) and (6) of the paper. Furthermore, it suffices to show the work for the case of labor type 1 in firm A, since the other three cases can be verified in a similar manner. Assume that the profit function is as follows:

\[ \Pi_A = f \left[ e\left(\frac{w_{1A}}{w_{1A}^*}\right)L_{1A}, e\left(\frac{w_{2A}}{w_{2A}^*}\right)L_{2A} \right] - w_{1A}L_{1A} - w_{2A}L_{2A} \] (1)

To maximize profits, we look at the partial derivatives of the profit function with respect to \( w_{1A} \) and \( L_{1A} \) and set these equal to zero.

\[ \frac{\partial \Pi_A}{\partial w_{1A}} = f'(*) e'(\frac{w_{1A}}{w_{1A}^*}) \left( \frac{L_{1A}}{w_{1A}^*} \right) - L_{1A} \] (2)

\[ \frac{\partial \Pi_A}{\partial L_{1A}} = f'(*) e\left(\frac{w_{1A}}{w_{1A}^*}\right) - w_{1A} \] (3)

Solving equations (2) and (3), we get

\[ e'(w_{1A}/w_{1A}^*) = e\left(\frac{w_{1A}}{w_{1A}^*}\right) \left( \frac{1}{w_{1A}^*} \right) \] (4)

Possible shapes for the effort equation are shown in Figures 3A-1 and 3A-2. Note that the Solow condition specifies a shape for the effort equation which includes a kink at the point where actual wage equals fair wage.
Figure 3A-1

Figure 3A-2
APPENDIX 3B

The following is the solution of the fair wage equations (equations (1) - (4) in Part V of the paper). Since we know the firms will pay the fair wage, we solve the equations with $w_{ij} = w_{ij}^*$. In an effort to simplify notation, we will drop the asterisks. The equations to solve are as follows:

\begin{align*}
  w_{1A} &= \beta_A w_{2A} + (1-\beta_A)w_{1B} \quad (1) \\
  w_{2A} &= \beta_A w_{1A} + (1-\beta_A)w_{2B} \quad (2) \\
  w_{1B} &= \beta_B w_{2B} + (1-\beta_B)w_{1A} \quad (3) \\
  w_{2B} &= \beta_B w_{1B} + (1-\beta_B)w_{2A} \quad (4)
\end{align*}

Substituting (1) into (2), (2) into (1), (3) into (4), and (4) into (3) generates equations (5), (6), (7), and (8), respectively.

\begin{align*}
  w_{1A} &= d_A w_{2B} + (1-d_A)w_{1B} \quad (5) \\
  w_{2A} &= d_A w_{1B} + (1-d_A)w_{2B} \quad (6) \\
  w_{1B} &= d_B w_{2A} + (1-d_B)w_{1A} \quad (7) \\
  w_{2B} &= d_B w_{1A} + (1-d_B)w_{2A} \quad (8)
\end{align*}

where

\begin{align*}
  d_A &= \beta_A (1-\beta_A)/(1-\beta_A^2) \\
  d_B &= \beta_B (1-\beta_B)/(1-\beta_B^2)
\end{align*}
Substituting (7) and (8) into (6), we get:

\[ w_{2A} \left[ d_A + d_B - 2 d_A d_B \right] = w_{1A} \left[ d_A + d_B - 2 d_A d_B \right] \]

\[ w_{2A} = w_{1A} \quad (9) \]

Substituting (9) into (7) and (8), we finally get the solution:

\[ w_{1A} = w_{2A} = w_{1B} = w_{2B} \quad (10) \]

Hence, wages will be equal for each type of worker, and wages will be equal across firms.
REFERENCES


