Leadership in the Repeated Prisoner’s Dilemma with Imperfect Monitoring and Communication

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Leadership has often been proposed as a tool to aid in problems of collective action and social interaction. This is further supported by the presence of strong leaders in various social movements throughout history. In this paper, I represent social interaction with an n-person prisoner’s dilemma, where monitoring is imperfect monitoring and there is some form of communication available. I will develop a formal theory of leadership that not only demonstrates the importance of leadership in supporting collective action and cooperation but also shows how collective action problems themselves might facilitate the development of the institution of leadership.

1. INTRODUCTION

The prisoner’s dilemma has been one of the most studied games in all of rational choice theory. In the standard two-person prisoner’s dilemma, each person has a choice to either cooperate or defect, and because of the structure of the payoffs, the unique Nash equilibrium of the game is for each player to defect. In the infinitely repeated version of this game where each player is aware of the other’s past actions, other equilibria emerge. Namely, by adopting certain “trigger strategies” there exists a Nash equilibrium where each player cooperates in every period.

This result is not only interesting but powerful in the analysis of our social institutions. The Hobbesian state of nature of self-interested individuals quickly devolves into a state of war, and demonstrates the difficulty in generating cooperative behavior among a group of self-interested individuals. There are also many real-world examples of problems in collective action. Ostrom (1994) discusses problems involving regulating commons, and specifically the inherent difficulty in managing our natural resources. Chong (1991) examines the difficulty in maintaining social movements. In each of these situations, the incentive problem of individuals has the structure of a prisoner’s dilemma. The study of the prisoner’s dilemma and its variants has lead us to a better understanding of collective action.

A major breakthrough in the study of social interaction was demonstrating that with repetition there exist equilibria to the prisoner’s dilemma where each player cooperates in every period. However, this “nice” equilibrium is heavily reliant on the assumption of perfect monitoring, whereby at the beginning of each period,

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each player receives a perfect signal of the play of the other players in all previous periods. Recent work in the general theory of discounted games has examined various relaxations of this assumption. The goal in this work has been to reestablish some version of the folk theorem,\(^3\) where every payoff vector that is feasible and individually rational is an equilibrium payoff when players are sufficiently patient or their discount rate is sufficiently small.

Similarly, the assumption of perfect monitoring even in a social context is too strong. In small, highly integrated groups, it is plausible that players are able to observe each other’s actions very closely. However, in larger groups and societies, it is generally unreasonable to assume that each person can observe everyone else.

In addition to repetition, another proposed solution to the collective action problem has been leadership. This idea is pursued by Wagner (1966) where he examines the integral role that leaders or “political entrepreneurs” play in facilitating collective action. Leaders facilitate cooperation by organizing, monitoring for “bad” behavior, and punishing deviants, given that they receive a certain benefit.

However, the formal literature of repeated games has not been able to incorporate leadership into collective action problems. Under the assumption of perfect monitoring, within the framework of the repeated prisoner’s dilemma, leaders are unnecessary. In order for leaders to be of any effect, we will require some imperfections in monitoring and some device by which leaders are able to “lead.”

In this paper, I examine a stylized version of the n-person prisoner’s dilemma with imperfect monitoring. The particular monitoring asymmetries are rather extreme by the standards of the current literature, and without any additional features to the model, there is no possibility of achieving an equilibrium that is “nice” in the sense that each player cooperates in every round. After that, I introduce leadership into the game by allowing for the possibility of communication. By introducing communication or a signaling period, I will show that “leadership” behavior will develop and allow us to achieve a “nice” equilibrium. In doing this, I will not only show how leadership may develop but will also show the power of communication in overcoming imperfections in monitoring. While communication or signaling in games with imperfect monitoring has been looked at, it remains a relatively unexplored area of repeated games especially in examining its relation to social contexts.

Implicit in this model is a relation between communication and leadership. While there are a variety of definitions of leadership, a certain natural leadership relation consists of a leader giving a subordinate instructions, and for the subordinate to follow those instructions. In order to replicate a similar relation, it is necessary to have some channel of communication or messaging available. In addition, leadership qualities are generally thought of as having an extensive social network, being able to express ideas and being of a certain social status – all of which imply a very sophisticated level of socialization, which may also be represented by his having a channel of communication with the other players.

In the following section, I examine the existing literature on games with imperfect monitoring and social theories of leadership. In section three I briefly describe the basic games and address the difficulties in the certain type of monitoring imperfection that I employ. In section four I examine the game without communication. In section five I specifically look at the game with communication leader, which has distinct properties not shared by the version with two or more leaders. Section six examines the game with more than one leader. Section seven includes conclusions.

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\(^3\)See Myerson (1991) 334 for definition of folk theorem or general feasibility theorem
and propositions for areas of further research.

2. LITERATURE REVIEW

2.1. Microeconomic foundations

There is an incredibly large body of work studying discounted repeated games and variations of the prisoner’s dilemma. One of the most important contributions to the field was Fudenberg and Maskin’s (1986) establishment of a general feasibility theorem, or “folk theorem” for repeated games with discounting. This result shows that any payoff vector that is both feasible and rational is an equilibrium payoff when discounting is sufficiently high.\(^4\)

These results however, are dependent on the assumption of perfect monitoring where in each period players receive a signal informing them of the play of all others in previous rounds. Recent work has studied various relaxations of this assumption. Earlier studies of imperfect monitoring concentrated on imperfect public monitoring. In this case, while players are not able to directly observe the actions of other players, each player receives a noisy signal of play in the previous rounds and this signal is assumed to be common knowledge among all of the players. Consider a game in which players work independently to prevent an accident. Each player can choose an effort level, but the players cannot observe one another’s effort levels. What is observable however is the occurrence of accidents, and this is public knowledge.\(^5\) The set of equilibria in these types of games can be characterized by using dynamic programming techniques as in Abreu, Pearce, and Stachetti (1990). Fudenberg, Levin, and Maskin (1994) establish a folk theorem for this type of imperfect monitoring, which showed that under certain conditions on the public monitoring technology all feasible and individually rational payoffs could be supported.

More recent work has focused on the problem of imperfect private monitoring. Like the case of imperfect public monitoring, players do not directly observe the actions of other players but receive only a noisy signal of the play in the previous periods. An example of this would be a market where firms do not observe each other’s prices and sales, but are able to observe their own sales, giving them a noisy signal about the play of others. Sekiguchi (1997) proves the existence of nearly efficient equilibria in the case where monitoring imperfections are small and players are sufficiently patient. Piccione (2002) and Ely and Valimaki (2002) establish partial folk theorems for the infinitely repeated two player prisoner’s dilemma with imperfect monitoring. Ely, Horner and Olszewski (2003) provide the sharpest results yet: they establish a set of equilibria which they call belief-free and characterize the set of payoffs for general two-person games with private monitoring, and these continuation strategies have the desirable robustness properties. These results however, are not rich enough to create a general folk theorem for repeated games with imperfect monitoring.

Some recent literature has taken on yet another approach to relaxing the assumption of perfect monitoring. Miyagawa, Miyahara, and Sekiguchi (2003) examine general repeated games where there is a cost to observation, and without paying this cost, players receive no information about the other player’s past actions. They

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\(^4\) Ely and Valimaki (2003)

\(^5\) For more details on this example, see Myerson (1991)
are able to prove an approximate folk theorem for several classes of repeated games (including the prisoner’s dilemma).

While by no means extensive, there is some literature that examines communication in repeated games and examines how imperfect monitoring can be overcome via communication. Kandori and Matsushima (1998) analyze a game where players receive diverse signals and players do not have a shared set of beliefs about what may have happened. In a game that is similar to my model, Ben-Porath and Kahneman (1996) consider repeated games in which each player only observes a subset of the other players.

There are slight differences between the game in Ben-Porath and Kahneman (1996) and the one that I will use. The communication period in the game I present happens at the beginning of each period. More interestingly, while the players in Ben-Porath and Kahneman communicate via public signals, in this game I assume that players signal one another privately. Also, by placing additional structure on the availability of information, I am able to find additional equilibria.

2.2. Cooperation and Theories of Leadership

Ostrom’s (1994) work on commons problems highlights a very important aspect of solving problems of collective action. With regards to use of natural resources, she shows that neither a market solution, nor a centralized state solution has proved very useful in overcoming commons problems. The most successful attempts at governing commons, according to Ostrom, are instances of self governance whereby cooperation is sustained by some sort of voluntary institution rather than a coercive power.

Axelrod, Taylor, and Hardin established using the repeated prisoner’s dilemma as a tool in analyzing the emergence of cooperation. Axelrod (1981) discusses the importance of the prisoner’s dilemma in political philosophy, international politics, and economic and social exchange. He concludes that players following a “tit-for-tat” strategy, or a strategy of reciprocity, will generally do better than any other strategy. Axelrod uses a repeated prisoner’s dilemma with pairwise competition between individuals, similar to the model that I will be using.

Bendor and Mookerjee (1987) expand on Axelrod’s analysis by no longer assuming that players monitor perfectly and that there is some “noise” in their observation. They show that as the “noise” increases, the value of individual strategies based on reciprocity decreases and centralized solutions are preferred. The also build a hierarchical model which combines aspects of both individualized strategies of reciprocity and centralized institutions.

Milgrom, North, and Weingast (1990) analyze a stylized version of the prisoner’s dilemma designed to represent anonymity in large decentralized groups. In this game, players are matched into pairs in each round to play a two-person prisoner’s dilemma. In each round, each player knows his own past moves and moves that other players chose when matched up with him. Milgrom, North, and Weingast then add in a central mediator who keeps track of players who have violated the cooperative norm. In the following model, I will use players similar to the central mediator, but assume a setting that has less information available to each player.

Wagner (1961), in a review of Olson’s work on collective action, notes the importance of leaders, or “political entrepreneurs” in facilitating collective action. Political entrepreneurs are “people who will pay the costs of soliciting and coordinating contributions in exchange for individual benefits such as power, prestige, or
a share of the profits derived from collective action.\textsuperscript{6}

There are various other works analyzing the type of people who become leaders. Mattozzi and Merlo (2006) propose that political leaders are drawn from neither “the best” nor “the worst” of what a country has to offer. Chong (1991), looks at the various traits and motivations of leaders that motivate collective action, and determines that leaders of collective action movements generally display some form of altruism.

Fiorina and Shepsle (1989) provide the most extensive formal analysis of political leadership. They postulate three theories of political leadership: the leader as an agent, the leader as an agenda setter and the leader as an entrepreneur. Of these three theories, the one that this paper will be most concerned with will be the leader as an entrepreneur facilitating collective action.

The largest concern of this paper however, will not be political leadership, nor will it be describing what types become leaders. The main concern of this paper will be recreating the pattern of leadership from a stylized but general model of social cooperation. Dion (1968) provides us with a useful criterion of the pattern of leadership. The pattern of leadership entails three components: i) a leader or team of leaders ii) the followers and iii) a functional relationship that exists between them. Of the three criteria, the third will be the most important to articulate. For this model the relation between leader and follower will be characterized such that:

i.) The leaders send out a “recommendation” about how to play based on previous play.

ii.) The followers follow the recommendation based on their beliefs about the leaders’ recommendation

\subsection{2.3. Additional Comments}

The origins of the particular prisoner’s dilemma I use can be found in a simulation I participated in during a seminar class. Part of this simulation involved a repeated prisoner’s dilemma between each of the players. This was designed to represent a very rudimentary form of trade, but can interpreted as simply as players having the choice to be “nice” or “mean” towards one another.

In the model I use in this paper, I borrow largely from the game used in the simulation combined with features used in Ben-Porath and Kahneman (1996). The basic model is an n-person pairwise prisoner’s dilemma where each player plays a two-person game with every other player. I elect to look at the case where communication is conducted privately at the beginning of each round. A subset of players have “monitoring privilege” where in each period they receive a perfect signal about the history of actions and payoffs. The rest of the players receive no signal about the history of play and do not even receive information about their own payoffs. I discuss this construction in the next section.

\textsuperscript{6}Chong (1991), 125.
3. THE GENERAL MODEL

3.1. The General Stage Game and Payoffs

Let there be a set $N = \{1, ..., n\}, n \geq 2$ of players. At stage $t$, each player $i \in N$ plays the following normalized prisoner’s dilemma with all players $j \in N_{-i}$:

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<th>$c$</th>
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<tr>
<td>$c$</td>
<td>1, 1</td>
<td>$-\epsilon, 1 + \gamma$</td>
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<tr>
<td>$d$</td>
<td>$1 + \gamma, -\epsilon$</td>
<td>0, 0</td>
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where player $i$’s action $s^t_{ij}$ can either be to cooperate, $c$, or to defect, $d$, where $\epsilon, \gamma > 0$. Alternatively, we may express the matrix of payoffs as a function $f : \{c, d\} \times \{c, d\} \rightarrow \mathbb{R}$ where

$$f(s^t_{ij}, s^t_{ji}) = \begin{cases} 
1 & \text{if } s^t_{ij} = c, s^t_{ji} = c \\
1 + \gamma & \text{if } s^t_{ij} = d, s^t_{ji} = c \\
-\epsilon & \text{if } s^t_{ij} = c, s^t_{ji} = d \\
0 & \text{if } s^t_{ij} = d, s^t_{ji} = d 
\end{cases}$$

The stage game payoff for a player $i$ in stage $t$ is a summation of the payoffs from the $n - 1$ two-person prisoner’s dilemmas in which $i$ plays: $\pi_i : \{c, d\}^n \rightarrow \mathbb{R}$ such that

$$\pi^t_i(s^t_{ij}, s^t_{ji} | j \in N_{-i}) = \sum_{j=1}^{n\setminus i} f(s^t_{ij}, s^t_{ji})$$

The utility for player $i$ is a discounted sum of his payoffs from each stage:

$$u_i(s^t_{ij}, s^t_{ji} | j \in N_{-i}, \forall t) = \sum_{t=0}^{\infty} (\delta)^t \pi^t_i$$

where $\delta \in (0, 1)$ is the discount factor which we will assume is the same among all players.\footnote{Note that in this equation, the superscript is used both as an index and an exponent. It will denote an exponent only when it is outside of parentheses. Otherwise, it is used as an index.}

3.1.1. The Stage Game for "leaders"

Let $L = \{1, ..., l\}$ be a set of $l$ players, $L \subset N$. At the beginning of each stage $t$, each player $i \in L$ knows the entire history of actions and payoffs.\footnote{In the formal theory of repeated games it is often modeled that players receive signals about play in the previous period. Since the structure of these signals is relatively simple in the case of both the leaders and followers, I will not forego using any additional notation to describe the information received at the beginning of each period.} Then, each player $i \in L$ plays the normalized prisoner’s dilemma with all players $j \in N_{-i}$, with the payoffs as described in equation 3.1 (1).

3.1.2. The Stage Game for "followers"

Let $F$ be the set of "followers" where $F = N \setminus L$. At the beginning of each stage $t$, each player $i \in F$ knows only of the history of his own actions. They do not receive any information regarding the actions of other players, nor do they receive
any information about their payoffs in previous periods.\textsuperscript{9} Then, each player plays the normalized prisoner’s dilemma as described in 3.1 (1).

3.2. Comments on the Model Structure

Several features of the game structure warrant additional explanation. The n-person prisoner’s dilemma I use is a pairwise prisoner’s dilemma played between \( n \) players. In other versions of the game, each player only has to make one decision about whether to cooperate or defect, and this decision goes towards one large social project. This game however, is focused more on the interaction that takes place between two players. This could actually be interpreted as more closely representing general social interaction since we may vary the way we behave among different players.

Another feature is that some players cannot observe the actions of other players at all, while some can observe them completely. It helps if we view acquiring information as very costly, perhaps costly enough where it is never worthwhile to pay the cost to monitor. And we can interpret those who are able to monitor freely either as exceptionally gifted, or more realistically, as “political entrepreneurs” who just enjoy working, or are altruistic. We may also view the group as highly separated or very large where actions are difficult to observe. We might imagine the leader as having a higher vantage point allowing him to see the other players. We may also consider the leader as the center of the network since he is the only one able to send out signals to the other players.

A more contentious assumption in the game is that certain players may not even observe their own payoffs. This assumption is also used in Miyagawa, Miyahara, Sekiguchi (2003). We can interpret this assumption by viewing the game as “finite” and that payoffs are received at the end of the game. We may view infinite-horizon games as a “long-term relationship” where no move is necessarily the last because no player knows what in which the game will end. Here, the discount factor \( \delta \) may also be interpreted as a continuation value of the game, which includes a probability of that game being the last.\textsuperscript{10}

Another possible interpretation which is more true to the origins of the game, is interpreting the cooperate and defect choices as votes of approval for another player, and these votes are made in secret. At the end of the game, payoffs are given based on the matrix in section 3.1. In this interpretation, the discount factor \( \delta \) is strictly interpreted as the continuation value based on the probability of the game ending. We may then interpret those who have monitoring privileges as agents of the players, and we may view them as being chosen by the other players.

3.3. Efficiency

In order to characterize and compare equilibria, I will often refer to the efficiency of an equilibrium. Since we are looking for equilibrium in which individuals cooperate, a welfare criterion such as efficiency is natural. The particular definition of efficiency that I use is that of Pareto optimality. In this particular game, inefficiency is characterized by any pair of players, \( i, j \) such that they are defecting

\textsuperscript{9}The signal received by followers is mostly a matter of convention. In none of the equilibria I discuss will this signal be of any importance. It is however, useful in interpretation, since it is difficult to justify a player not remembering his own action from the previous period.

\textsuperscript{10}See Myerson (1991) 308 for more on the interpretation of the discount factor.
against each other in the same period, that is for some $t$, $s^t_{ij} = s^t_{ji} = d$. An equilibrium is efficient if it is not inefficient. We could have a slightly stronger version of inefficiency, but for the class of behavioral strategies that I consider, this definition will suffice. In all cases I consider, the standard definition of Pareto optimality will also apply when I consider efficient equilibria.

4. NO COMMUNICATION

Consider the game where no communication takes place, following the game outlined in section 3.1. There are a set of players $N = \{1, ..., n\}, n \geq 2$. A subset of players, know the the entire history of actions and payoffs. The rest of the players are only informed of the history of their own play in the previous rounds. In each period, a player plays a two-person prisoner’s dilemma with each of the $n - 1$ other players, with the payoffs as described in 3.1 Each player’s utility is a weighted sum of the payoffs from the individual games as denoted in section 3.1 (3) and 3.1 (4).

We can characterize all the equilibria in the case where there are no leaders, $L = \emptyset$, or there is one leader, $L = \{1\}$. In the case where $L = \emptyset$, it is clear that since no one observes any actions, the unique strategies in equilibrium must be s.t. $s^t_{ij} = d$ for all players $i \in N, j \in N_{-i}$ in every period $t$. Alternatively we may think about this as a one-shot game since players receive no information other than their own actions in each round. So the unique equilibria will be the stage-game equilibria in every period.

In the case where $L = \{1\}$, since player 1 is unobserved by anyone else, his unique strategy in equilibria will be to always play $s^t_{1j} = d$ for all $j \in N_{-1}$ in every period $t$. For every other player $i \in N_{-1}$, since there is a unique player monitoring, and that player will play $d$ regardless of any action $i$ takes, the unique action in equilibrium is for him to play $s^t_{ij} = d$ for all players $i \in N, j \in N_{-i}$, in every period $t$ as well.

In the case where there are multiple leaders, the set of equilibria is not as simple. But making some assumptions about the structure of payoffs, we may find certain properties common to all the equilibria.

**Proposition 1.** Let $n \geq 3$ denote the number of players in the game, and $l$ denote the number of leaders in the game where $2 \leq l < n$. Then there exist no efficient equilibria in the game without communication if $(n - l - 1)\gamma > l\epsilon$ and $n - l > 1$.

As we can see, the game in which no communication takes place makes it very difficult to find a “nice” or first-best equilibrium in which everyone cooperates in every period along the equilibrium path of play. The problem stems from the position of those who cannot monitor, as the only strategy for them to play in equilibrium based on the information they have available to them is to defect in every period.

One possibility to overcome this relatively severe imperfection in monitoring is to introduce some form of communication between those who are able to monitor and those who are unable to monitor. I model this by allowing those who are monitors or “leaders” to send private signals to each of the other players.

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$^{11}$See Appendix for proof
5. COMMUNICATION AND ONE LEADER

5.1. The Game Structure

I introduce communication into the game by first examining the game with one leader, or one person with monitoring power. Also, the ability to send out signals is reserved for this player.\(^{12}\) In this section I will assume that there is 1 leader, \(L = \{1\}\), and that communication exists, where I may send out a signal to each of the other players. There are a set of players \(N = \{1, \ldots, n\}, n \geq 2\). In each stage, each player plays the two-person prisoner’s dilemma with each of the other \(n - 1\) players with the payoffs as described in section 3.1 equation (1). The utility of a player \(i\) is the weighted sum of his stage game payoffs as in equation (3). In order to establish efficient equilibria, I resort to a more complex notation for messages which I will simplify in later sections.

5.1.1. The Stage Game for "leaders"

Let \(L = \{1\}, L \subset N\). At the beginning of each stage \(t\), player 1 is informed of the entire history of actions and payoffs. Then, 1 sends a private signal \(M^t_1\), an \(n \times n\) matrix where each entry, \(m^t_{1ij} \in \{c, d\}\). Then 1 plays the two person prisoner’s dilemma with each player \(j \in N_{-1}\) as described in section 3.1 equation (1).

5.1.2. The Stage Game for "followers"

At the beginning of each stage \(t\), each player \(i \in F\) only has information regarding the history of his own actions. The followers do not receive any information regarding the actions of other players, nor do they receive any information about their payoffs in previous periods. Then, \(i\) receives a signal \(m^T_{1i}\), where \(m^T_{1i}\) is the \(i\)th row of \(M^t_1\). Then each player plays the normalized prisoner’s dilemma as described in 3.1 (1).

5.2. Efficient equilibria

Since 1 is not observed by anyone, it is impossible to construct an equilibrium where every player \(i \in N\) plays \(s^t_{ij} = c\) in every period \(t\) for all players \(j \in N_{-i}\). However, it is possible to construct an equilibrium where every player \(i \in N_{-1}\) plays \(s^t_{ij} = c\) in every period \(t\) for all \(j \in N_{-i}\).

**Proposition 2.** For \(N = \{1, \ldots, n\}, n > 3\) and \(\delta > 1 - \frac{1}{1 + \gamma}\), an efficient equilibrium exists in the repeated prisoner’s dilemma with imperfect monitoring and one leader with communication.\(^{13}\)

A formal proof is given in the appendix. The intuition behind the proposition is the following set of strategies. The leader, 1 sends a message, where he tells everyone to vote for someone if he has followed his directions in all previous periods, and if a player \(j\) has not followed his directions, he tells all players to defect against \(j\). The leader always defects against all followers. The strategy for followers is to follow the directions of the leader. The proof that these strategies form an equilibrium, and that the result is efficient is shown in the appendix. Unfortunately, this

\(^{12}\)This assumption is not necessary; we may extend the ability to communicate to all players and assume that messages from non-leaders are "noise"

\(^{13}\)See Appendix for proof
set of trigger strategies is not robust to equilibrium refinements such as sequential rationality.

With these strategies, the leader has an incentive to report deviations made by the followers. Given these strategies, if a player defects against any player, his best response strategy in all future periods is to defect against everyone, including the leader. Since the follower $j$ who defects will defect in every future period anyway, the leader will report this defection to all the other followers and direct them to defect against $j$. If there are enough followers who are still cooperating, the payoff to each follower from cooperating exceeds the payoff from defecting. In this equilibrium, all followers cooperate with the leader even though the leader defects against everyone else since the gains from cooperating with the followers exceeds the cost of cooperating with the leader.

Notice however, that there is a problem when there are only two followers. Suppose there are two followers, and one of them, say $j$, defects against everyone. If the leader, $1$, reports this to the other follower $i$, then $i$ knows that in every subsequent period, no player will be cooperating with him. Therefore, his unique best response is to defect against everyone. Therefore, given a defection by $j$, it is not a best response for $1$ to report. It is because of this "subgame" that we require that $n > 3$, and this subgame is also the reason that this equilibrium result is not robust to equilibrium refinements.

While there are many equilibria to the game described in this section, ranging from the horribly inefficient case in which everyone defects to the equilibria described above, I do not attempt to specify the entire set of equilibria. However, the equilibrium described in this section has interesting properties and it warrants some discussion.

First of all, this equilibrium has the desired properties of a leadership relation. The leaders and followers are evident. The relationship between them is such that the leader reports if anyone (besides himself) has defected, and sends a recommendation telling people whether to cooperate or to defect. The followers believe his information, and match their play to the leader’s message.

Also interesting in this particular equilibrium is the way in which the leader is able to extract rents from the followers. The leader defects against everyone, but all the followers are required to cooperate with him. This allows us to interpret the leader as a political entrepreneur who facilitates collective action and in turn receives a larger share of the benefits made possible by the collective action.

While the bounds on the discount factor might seem a bit complicated, there is a nice interpretation. The value of cooperating relative to defecting is a function of the discount factor, the one-shot benefit of defecting, and the dues that are given to the leader each period.

We are able to see a significant difference between the game with one leader with and without communication. Without communication, we have shown that the unique equilibrium was for players to defect in every period. With communication, however, it is possible not only to construct an equilibrium where some players cooperate, but to construct an equilibrium that is efficient, and “almost” first best.

\[^{14}\text{This is not a true subgame since a subset of players do not have full information.}\]
6. COMMUNICATION AND TWO OR MORE LEADERS

6.1. The Game Structure

In this section, I extend the monitoring and communication powers to a set of more than one persons \( L = \{1, \ldots, l\} \), and assume that communication exists, where \( i \in L \) may send out a signal to each of the other players. There are a set of players \( N = \{1, \ldots, n\}, n \geq 2 \) and a set \( L \subset N \) of leaders where \( L = \{1, \ldots, l\}, l \geq 2 \). In each stage, each player plays the two-person prisoner’s dilemma with each of the other \( n - 1 \) players with the payoffs as described in section 3.1 (1). The utility of a player \( i \) is the weighted sum of his stage game payoffs (3).

6.1.1. The Stage Game for "leaders"

Let \( L = \{1, \ldots, l\} \) be a set of \( l \) players, \( L \subset N \). At the beginning of each stage \( t \), each player \( i \in L \) knows the history of actions and payoffs from the previous rounds. Then, \( i \) sends a private signal \( M^t_i \) an \( n \times 1 \) matrix, where the entries \( m^t_{ij} \in \{c, d\} \) for all \( j \in N \). \( i \) then receives messages \( m^t_{ji} \) from players \( j \in L_{-i} \). Then \( i \) plays the prisoner’s dilemmas described in 3.1 (1).

6.1.2. The Stage Game for "followers"

At the beginning of each stage \( t \), each player \( i \in F \) is only informed of the history of play of his own actions. The followers do not receive any information regarding the actions of other players, nor do they receive any information about their payoffs in previous periods. Then, \( i \) receives a signal \( m^t_{ji} \) from players \( j \in L \). Then \( i \) plays the prisoner’s dilemmas as described in 3.1(1).

6.2. Efficient Equilibria

With two leaders, we may find equilibria where each player cooperates in every period. In the previous section, the problem with only having one monitor or leader is that he will never cooperate because his own actions are not monitored. With multiple leaders however, trigger strategies can once again be used, this time to coax the leader into cooperating.

**Proposition 3.** For \( \delta > 1 - \frac{1}{1+\gamma} \), there exists a sequential equilibria to the prisoner’s dilemma with imperfect and monitoring where in every period, each player cooperates along the equilibrium path of play.\(^{15}\)

A formal proof is given in the appendix, however, the intuition behind the set of strategies is rather simple and is only made difficult by the notation. Let the strategy for the leaders be such that if anyone defects in any period, in their message they tell everyone to defect. Then they will defect against everyone. If, on the other hand, everyone has cooperated in every period, they will send a signal telling everyone to cooperate and they will themselves cooperate. The follower will cooperate as long as every signal that is received tells them to cooperate. Otherwise, they will defect against everyone. In order to show that these strategies satisfy the conditions for sequential equilibria, we need to specify a set of beliefs for the followers. However, given their relatively simple strategies, the set of beliefs is very simple.

\(^{15}\)See Appendix for proof
So long as the leaders are abiding by the given strategies, it is clear that the strategies for the followers form a best response. The important thing to show is that the leaders will actually report defections. But for any given leader i, it is simple enough to see that if someone defects, since all the other leaders will report the defection, given that followers will only cooperate if all the messages they receive tell them to cooperate, it is a best response for him to report as well.

The set of equilibria in the game with multiple leaders is very large and I dare not attempt to specify all of them. Even the set of equilibrium payoffs is very large since there are multiple leaders both who are able to monitor perfectly, Ely and Valikami (2001) showed that for any set of individually rational payoffs there exists an equilibrium with those payoffs. In this game the set of equilibrium will necessarily be much larger than the set that Ely and Valimaki describe since there are many two-person games being played.

That we are able to construct equilibrium in which players cooperate in every period is an interesting finding. This builds on the work of Ben-Porath and Kahneman (1996) and is able to establish a similar (though far less robust) result to theirs even when signals are private.

We are also able to satisfy the requirements for a leadership relationship. The leaders and followers are once again obvious. In this relationship, the leaders, so long as they are coordinated in their signals, make recommendations to followers about what actions to take. The followers, believing the information in the signal, then follow the recommendation. In this particular equilibrium however, the leaders do not receive any “extra” benefits for their efforts.

Perhaps even more interesting is the comparison between this result and our results from the one-person game. With multiple leaders, it is possible for the leaders to “check” one another, and thus induce cooperation from everyone. This can be seen as an argument about how it is better to have more people as leaders, in order to have them serve as “checks and balances” towards each other, and perhaps an argument against having a single person in charge.

Another interesting feature of this equilibria is the way in which the “followers” process the signals they receive from the leaders. If the signals are not coordinated, then they automatically assume the worst. This belief systems of the followers makes high demands on the consistency of the signals that are sent by the leader.

There are also equilibria that follow from the game with one leader and communication where the leaders are able to extract rents from the followers. These equilibria are also efficient and are able to capture the sense of leaders receiving benefits for their extra effort.

Also, by allowing the signal that is sent by the leaders to be more sophisticated, we may find equilibria that are a bit more complex and more intuitive than the one described above. The previous equilibrium I mentioned is a very strict analogue of the two-person grim trigger strategies – if anyone defects at any time, everyone will defect against everyone else for the remaining periods. We can construct equilibria whereby if a follower defects, he is the only one punished.

\[16\text{See Appendix, Example 1}\]
\[17\text{See Appendix, Example 2}\]
7. CONCLUSION

In this paper I have developed a theory of leadership based on collective action problems and imperfections in monitoring. In large, disaggregated groups, members are often not able to observe one another. I show that if even one person has the ability to observe the behaviors of others and if he is able to communicate with the other members, then cooperation is still possible. The behavior that is exhibited in several of the equilibria satisfies the requirements of a leadership relationship.

One of the interesting results of this approach is that I have been able to create an account of leadership from "within" the structure of the prisoner’s dilemma. The leaders resemble a mechanism, sending out an appropriate signal based on the play of others. However, unlike a mechanism, the leader is actually a participant in the game facing his own incentive problems. In effect, there are two main results of this paper: first, we are able to show how leadership may allow us to overcome collective action problems, and second, we are able to simultaneously show that leadership may actually develop from collective action problems.

These results allude to Ostrom’s ideas of self-governance. The outcomes in this model are not dependent on market mechanisms or a centralized power - cooperation is sustained by the actions of the individuals involved, and in the process, a new institution, namely leadership, emerges.

The leaders that emerge in this model have interesting roles to play. Their message gives information to the other players, and their message is also makes a recommendation about how everyone should play. The leaders also implicitly play the role of a coordinator. This is evidenced by uniformity of the messages they send out. With multiple leaders, the importance of coordination becomes increasingly important as the follower’s cooperation depends on coordination between the messages of the leaders. Leaders also have the role of punishing deviators. They not only punish by their own defection, but because they have influence over the decisions of others, they are able to enforce group punishments against deviators.

While some of the assumptions, especially the ones concerning the information structure, are limiting and perhaps extreme, they highlight the vast difference that leaders can make. Beginning with a setting where no cooperation is remotely possible, if we introduce one person who has "a better vantage point" and is able to talk to other people, then suddenly, all of those people are able to cooperate with one another.

We may also view the leader positions as elected positions since those individuals have powers and abilities that the rest do not have. In this case, from a design perspective, it would be preferable to have multiple leaders rather than a single leader. While there are some addition coordination issues with multiple leaders, the advantage is that the leaders are able to monitor one another. In this case, it serves as a very rudimentary argument for separation of powers if the rent being extracted by a single leader is too high.

Most of the strategies that I have considered have been variations on trigger strategies. This is intentional, not only for their simplicity, but because they represent a behavior of reciprocity and are most useful and reasonable when thinking about general social interaction. While mixed strategies are also viable in repeated games, when considering problems of cooperation and collective action, behavioral strategies such as grim trigger, or tit-for-tat are more intuitive to use. In further studies of this model it would be useful to look at other punishment strategies. Trigger strategies represent a maximal punishment - in examining social institu-
tions, it would be useful to include the possibility of renegotiation.

In further studies, I would like to relax the assumption that some people receive no information, and instead assume that there is a cost to acquiring information for all players. In this setting, it would be interesting to see whether or not communication can generate an equilibrium that is "less costly" where cooperation is made possible with less people paying the cost to learn the history of play. I would also like to change the game from analyzing individual prisoner’s dilemma to a more traditional n-person prisoner’s dilemma, since it might represent a social good more accurately.

Another possible extension of this work is a more robust theory of the repeated games with imperfect monitoring and private messages. In this paper I use the very particular case when one or more players have full knowledge of the history of play and the rest have none. It would be interesting to examine if in this more robust setting that cooperation would be possible in the prisoner’s dilemma, or to determine what sort of minimal requirements are necessary in order to achieve cooperation.

A natural extension would be to integrate concepts of networking into the model. In this model, the leader is networked to everyone, and none of the followers have connections between each other. It would be interesting to examine other networking structures to see if it is possible to generate cooperation in a less centralized setting.

8. **APPENDIX**

**Proof of Proposition 1**

To show that no efficient equilibria exist, it is enough to show that in any equilibria, there exists a pair of players, \( i, j \) such that each is defecting against the other. I will show that under the specified payoffs, no follower will ever vote for another follower. Since we assume that there is more than one follower, no equilibria will be efficient.

Suppose each \( i \in F \) is cooperating with every \( j \in F, j \neq i \). By defecting against all followers, \( i \) can gain \((n - l - 1)\gamma\). Assuming that \( i \) defects against everyone, the maximum punishment that can be enforced against \( i \) is for all of the leaders to defect against him forever. Thus, he stands to lose \( le \) in every period. Assuming that \((n - l - 1)\gamma > le\), we see that in no equilibrium is it rational for \( i \) to cooperate with the other followers.

This proposition could be generalized, but these particular limits provide a very simple proof for inefficiency. We cannot immediately assume that no follower will be will to cooperate with the other followers since the leaders are still able to punish the followers, even though they are unable to observe in future rounds. If the punishment they are able to administer is large enough, it is possible that efficient equilibria (even sequential) can be found if \( \delta \) is significantly high.

**Proof of Proposition 2**

Consider the following set of strategies. In period \( t \), let 1 construct his message by the following process. Let \( m^t_{11} = c \) for all \( i \in N \). For \( m^t_{1ij} \) where \( j \neq 1 \), let

\[
m^t_{1ij} = \begin{cases} 
  c & \text{if } \forall t' < t, k \in N, s^{t'}_{jik} = m^t_{1jk} \\
  d & \text{if } \exists t' < t, k \in N : s^{t'}_{jik} \neq m^t_{1jk}
\end{cases}
\]

I will frequently use this abuse of notation where the set \( N \) is used interchangeable to describe both the set of players and a set of indices used for the message.
Then, 1 defects against everyone, \( s'_{1i} = d \) \( \forall i \in N_{-i} \). The strategy for \( i \in F \) is to set \( s'_{ij} = m'_{1ij} \) for all \( j \in N_{-i} \).

To show that these payoffs are individually rational, it suffices to show that the payoffs are rational for the followers. With the proposed strategies, the payoff to each player is \( \frac{1}{1 + \gamma} \), since in \( n - 2 \) of the games, each player is cooperating, and in one game, he cooperates, while the other player defects. Not adhering to this strategy, the best a player could do would be to defect against everyone (since if he defects against anyone, no one will cooperate with him in subsequent periods). Thus, his payoffs would be \( (n - 2)(1 + \gamma) \). Rationality is satisfied if \( \delta > 1 - \frac{(n-2+c)}{(n-2)(1+\gamma)} \geq 1 - \frac{1-\gamma}{1+\gamma} \) since \( n > 3 \) and \( \gamma > 0 \).

Since we have assumed that \( n > 3 \), if any one follower, \( i \), decided to defect, it would be a best response for the leader, 1 to report it. Since we can assume that all the remaining followers are abiding by the strategy, the payoff from abiding by the strategy is greater than the payoff from defecting.

Efficiency is satisfied, since any change strategy will lower the expected payoff of at least one player. If 1 cooperates with anyone, his payoffs would be strictly lowered by \( \gamma \). If any follower defects, his payoff is increased by \( \gamma \), but he also lowers another player’s payoffs by \( 1 + \epsilon \). This can also be seen since there is no situation where two players are defecting against each other and their payoffs could be strictly increased by cooperating with one another.

**Proof of Proposition 3**

Consider the following set of strategies. In period \( t \), \( i \in L \) constructs his message \( m^t_i \) as follows:

\[
 m^t_{ij} = \begin{cases} 
 c \forall j \in N & \text{if } \forall t', t, k, l \in N, s^t_{kl} = c \\
 d \forall j \in N & \text{if } \exists t', t, k, l \in N : s^t_{kl} \neq c 
\end{cases}
\]

\( i \) also adopts a trigger strategy in each of the two-person games in which he plays:

\[
 s^t_{ij} = \begin{cases} 
 c \forall j \in N_{-i} & \text{if } \forall t', t, k, l \in N, s^t_{kl} = c \\
 d \forall j \in N_{-i} & \text{if } \exists t', t, k, l \in N : s^t_{kl} \neq c 
\end{cases}
\]

In period \( t \), for \( i \in F \), let \( i \)'s beliefs be the conditional probabilities:

\[
\mu(\forall t' < t, j, k \in N, s^t_{kl} = c | \forall j \in L, m^t_{ji} = c) = 1 \\
\mu(\exists t' < t, j, k \in N : s^t_{kl} \neq c | \exists j \in L : m^t_{ji} \neq c) = 1
\]

\( i \) also adopts the following trigger strategy:

\[
 s^t_{ij} = \begin{cases} 
 c \forall j \in N_{-i} & \text{if } \mu(\forall t' < t, k, l \in N, s^t_{kl} = c) = 1 \\
 d \forall j \in N_{-i} & \text{if } \mu(\exists t' < t, k, l \in N : s^t_{kl} \neq c) = 1 
\end{cases}
\]

To show that these payoffs are rational or incentive compatible, note that the "maximal defection" is for a player to defect against everyone. The payoff for this defection is one period gain, \( (n - 1)\gamma \). The payoff gained from adopting the previous strategy is \( \frac{n-1}{1+\gamma} \). Therefore, for \( \delta > 1 - \frac{1}{1+\gamma} \), the strategies are incentive compatible.

In order to show that these strategies form a sequential equilibrium, it is enough to show that they satisfy sequential rationality.\(^{20}\) Sequential rationality is also

\(^{19}\)During any given period, these payoffs will be scaled down by a factor that depends on the period of the defection.

\(^{20}\)For a description of applying sequential equilibrium to infinite games, see Ben-Porath and Kahneman (1996)
satisfied quite trivially. Consider the case for a follower. For any \( i \in F \) in period \( t \), if any signal \( m_{ij}^t = d \), his beliefs infer that some person has defected. Given the strategies of all other players, they will all defect in every subsequent period, so \( i \)'s best response is to defect in every subsequent round as well.

Consider the case of a leader. If \( i \in L \) observes that someone has defected in any period, he is indifferent between sending \( M_{ik}^t = d \) and \( M_{ik}^t = c \) to all \( k \in N - i \), since \( k \in L - i \) will send out \( M_{jk}^t = d \), and based on the beliefs of the followers, they will always defect regardless of the signal that \( i \) sends out. Also, based on this signal, the strategy of the followers will be to defect in every subsequent period, and the strategy of the leaders is to defect in every period, so \( i \)'s best response is to defect in every subsequent round as well.

**Example 1. Equilibrium where Leaders Extract Rents**

In this example, we assume the same structure as in section 6. Let \( L = \{1, ..., l\} \) be a set of \( l \) players, \( L \subset N \). At the beginning of each stage \( t \), each player \( i \in L \) knows the history of actions and payoffs from the previous rounds. Then, \( i \) sends a private signal \( M_i^t \) an \( n \times 1 \) matrix, where the entries \( m_{ij}^t \in \{c, d\} \) for all \( j \in N \). \( i \) then receives messages \( m_{ji}^t \) from players \( j \in L - i \). Then \( i \) plays the two-person prisoner’s dilemmas described in 3.1 (1). At the beginning of each stage \( t \), each player \( i \in F \) is only informed of the history of play of his own actions. They do not receive any information regarding the actions of other players, nor do they receive any information about their payoffs in previous periods. Then, \( i \) receives a signal \( m_{ji}^t \) from players \( j \in L \). Then \( i \) plays the prisoner’s dilemmas as described in 3.1(1). I will only specify what is necessary for the strategies to constitute a Nash equilibrium, however, a rather elementary belief system similar to that used in section 6 can be used to generate a sequential equilibrium.

Consider the following set of strategies. Suppose all \( i \in L \) adopt the following strategy where in every period \( t \), \( i \) constructs his message \( M_{ij}^t \) such that:

\[
M_{ij}^t = \begin{cases} 
  c & \text{if } j \in N \text{ and } \forall t' < t, \forall k, l \in F, s_{kl}^{t'} = c, \forall k, l \in L, s_{kl}^t = c \\
  d & \text{if } j \in N \text{ and } \exists t' < t, \exists k, l \in F : s_{kl}^{t'} \neq c \cup \exists k, l \in L : s_{kl}^t \neq c
\end{cases}
\]

\( i \) adopts a trigger strategy regarding his actions in the two-person games

\[
s_{ij}^t = \begin{cases} 
  c & \text{if } j \in L - i, d & \text{if } j \in F \text{ and } \forall t' < t, \forall k, l \in F, s_{kl}^{t'} = c, \forall k, l \in L, s_{kl}^t = c \\
  d & \text{if } j \in N - i \text{ and } \exists t' < t, \exists k, l \in F : s_{kl}^{t'} \neq c \cup \exists k, l \in L : s_{kl}^t \neq c
\end{cases}
\]

In period \( t \) for \( i \in F \), let him play "cooperate" with everyone if he receives a signal telling him to cooperate from all the leaders, and let him defect if any signal receive tells him to defect.

\[
s_{ij}^t = \begin{cases} 
  c & \text{if } j \in N - i \text{ and } \forall k \in L, m_{ik}^t = c \\
  d & \text{if } j \in N - i \text{ and } \exists k \in L : m_{ik}^t \neq c
\end{cases}
\]

For \( \delta > \max\{1 - \frac{n-l-1}{(n-l-1)(1+\gamma)}, 1 - \frac{(n-l)(1+\gamma)+l-1}{(n-l)(1+\gamma)}\} \) where \( n - l - 1 - \delta c > 0 \), this set of strategies forms an equilibrium, and with the addition of beliefs, a sequential equilibrium. To show that the payoffs are individually rational, consider first the case for \( i \in L \). Playing the specified strategy, the expected payoffs for each round is \( l - 1 \) from cooperating with the other leaders and \( (n - l)(1+\gamma) \) from defecting against the followers. The total payoff then is \( (n-l)(1+\gamma)+l-1 \). The only way to do
better would be to defect against the leaders, and then the value of all subsequent periods would be 0. So the maximum benefit from defecting, would be the one period gains from defecting, $(n - 1)(1 + \lambda)$. By our choice of $\lambda$, we see that the payoffs are rational.

Consider the case of $i \in F$. The per period payoffs using these strategies is the $n - l - 1$ gained from cooperating with the other followers combined with the loss from cooperating with the leaders who are defecting. So the total payoff for the entire game is $n - l - 1 - l\epsilon$. If $i$ plays any other strategy, all the other players will defect based on their strategies. So the maximum gain from defecting is the one period gain of $(n - l - 1)(1 + \gamma)$. By our choice of $\delta$ and since $n - l - 1 - l\epsilon > 0$, rationality is satisfied.

To show that this set of strategies would satisfy sequential rationality is quite trivial. For any $i \in L$, at any information set, it is a best response to report as specified in the equilibrium because all others will report the same way, and there is no gain to sending a contradicting signal. Also, if any leader defects against a leader or a follower defects at all, it is a best response to defect against everyone, since everyone else will defect in every subsequent period based on their strategies. For any $i \in F$, observing any signal $M_{ji}^t = 0$, $i$ believes that someone has not played a strategy in equilibrium and that based on the given strategies, everyone will defect in the next period. So $i$’s best response is to defect against everyone as well.

That this equilibrium shows leaders extracting rents from the other players is quite clear. Instead of gaining $n - 1$ every round with the strategies specified in section 6.2, each leader gains $(n - l)(1 + \gamma) + l - 1$ per round. This comes at the expense of the followers, whose payoffs are necessarily reduced. Interestingly, this set of payoffs is also efficient, because there are no pair of players $i, j$ such that $i$ and $j$ are defecting against one another.

**Example 2. Equilibrium where Leaders Punish Individuals**

In this example, we assume a structure largely similar to the one used in section 6, except that the structure of the signals sent to the followers is modified. Let $L = \{1, ..., l\}$ be a set of $l$ players, $L \subset N$. At the beginning of each stage $t$, each player $i \in L$ knows the history of actions and payoffs from the previous rounds. Then, $i$ sends a private signal $M_i^t$ an $n \times n$ matrix, where the entries $m_{ijk}^t \in \{c, d\}$.

$i$ then receives messages $m_{ji}^t$, the $i$th row of $M_{ji}^t$, from players $j \in L \setminus i$. Then $i$ plays the two-person prisoner’s dilemmas described in 3.1 (1). At the beginning of each stage $t$, each player $i \in F$ is only informed of the history of play of his own actions. They do not receive any information regarding the actions of other players, nor do they receive any information about their payoffs in previous periods. Then, $i$ receives a signal $m_{ji}^t$ from players $j \in L$. Then $i$ plays the prisoner’s dilemma as described in 3.1(1). Once again, I will only specify what is necessary for the strategies to constitute a Nash equilibrium, although an extension to sequential rationality is not difficult.

Consider the following set of strategies. Suppose $i \in L$ constructs his message $M_i^t$ by telling people to defect against everyone if any leader does not follow his recommendations. If all the leaders have followed his directions, then he tells others defect against an individual follower if that follower has not followed his recommendations, and to cooperate if they have. That is,

$$m_{ijk}^t = d \text{ if } \exists t' < t, l \in L, l' \in N: s_{l,l'}^{t'} = m_{l,l'}^{t'}$$
If, on the other hand, $s_{t}^l = m_{t}^l$ for all $t < t, l \in L, l' \in N$, for any $k \in N$,

$$m_{t}^i_{jk} = \begin{cases} 
    c & \forall j \in N \text{ if } \forall t' < t, l \in N, s_{t}^l = m_{t}^l \\
    d & \forall j \in N \text{ if } \exists t' < t, l \in N : s_{t}^l \neq m_{t}^l
\end{cases}$$

The trigger strategy that $i$ follows is also a bit more complex than the strategies used in the previous sections, since he uses different strategies depending on whether the defection is made by a leader or a follower. If a follower does not follow directions, then he only punishes that individual follower. If a leader does not follow directions, he defects against everyone.

If $s_{t}^l = m_{t}^l \forall t' < t, l \in L, l' \in N$,

$$s_{t}^l = \begin{cases} 
    d & \forall j \in N \text{ if } \exists t' < t, l \in L, l' \in N : s_{t}^l = m_{t}^l
\end{cases}$$

The strategy for $i \in F$ is to follow the direction of the leaders as long as they are the same. Otherwise, $i$ will defect. In period $t$, $i$ plays

$$s_{t}^l = \begin{cases} 
    m_{t}^i_{kij} & \text{if } m_{t}^i_{kij} = m_{t}^i_{lij} \forall l \in L, l \notin k \\
    d & \exists l, l' \in L : m_{t}^i_{lij} \neq m_{t}^i_{lijk}
\end{cases}$$

For $\delta > 1 - \frac{1}{1+\gamma}$ these strategies form a Nash equilibrium.\textsuperscript{21}

To show that these strategies form a Nash equilibrium, we need only show that these payoffs are individually rational. Consider any deviation in the strategy of the follower. If the follower defects, no one will cooperate with him in the following period because he will be reported. Thus the maximum payoff from defecting is $(n - 1)(1 + \gamma)$. The payoff from adhering to the strategy is $\frac{n(\gamma^2 + \gamma - 1)}{\gamma}$. Thus for $\delta > 1 - \frac{1}{1+\gamma}$, the followers will adhere to the strategy. This also holds for any deviation in the actions for leaders, and any deviation in the messages sent by a leader can only lead to strictly worse payoffs.

In this particular equilibrium, if a follower defects, he is singled out for group punishment. It is not difficult to show that this satisfies sequential rationality since we have multiple leaders monitoring one another. However, the stronger punishment for a defection by a leader is necessary to avoid the "subgame" when there is only one leader adhering to the strategy.

REFERENCES


\textsuperscript{21}It is not difficult for these strategies to satisfy stronger conditions of a sequential equilibrium by specifying a set of beliefs for the followers.


